Specific Heats Problem 1

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Abstract

Here we treat a problem in specific heat as a mere algebra word problem.

1 Problem Statement

A 100 g gold-copper alloy sample has its temperature raised 23.4 °C by adding to it 200 calories of heat. The owner of the sample was told that the amount of copper in the sample is less than 50% by weight. Is this claim true or false?

2 Setup

Let Q_x be the heat added to a substance of mass m_x and specific heat C_x. The *specific heat* of a substance is the amount of heat energy needed to raise the temperature of one gram of the substance 1°C. The equation that relates these thermodynamic variables for element x is

$$Q_x = m_x C_x \Delta T\,, \tag{1}$$

where T is the temperature in kelvin or °C. Either scale will work because ΔT deals only in temperature differences. So, for gold (Au) and copper (Cu), we have that

$$Q_{\mathrm{Au}} = m_{\mathrm{Au}} C_{\mathrm{Au}} \Delta T \quad \text{and} \quad Q_{\mathrm{Cu}} = m_{\mathrm{Cu}} C_{\mathrm{Cu}} \Delta T\,, \tag{2}$$

where

$$C_{\mathrm{Au}} = 0.0301\,\mathrm{cal/g\cdot K} \quad \text{and} \quad C_{\mathrm{Cu}} = 0.0923\,\mathrm{cal/g\cdot K}\,. \tag{3}$$

3 Scheme & Solution

Are there any totals in the problem? (Almost always there are.)

First, there is a total of 100 g of alloy, which is in gold and copper. By additivity of mass, the total mass is the sum of its parts:

$$100 = m_{\mathrm{Au}} + m_{\mathrm{Cu}}\,. \tag{4}$$

And, of course, there's the total heat applied to the sample, and it is also additive in its distribution between the gold and copper (in thermal equilibrium).

$$\begin{aligned} 200 &= Q_{\mathrm{Au}} + Q_{\mathrm{Cu}} \\ &= m_{\mathrm{Au}} C_{\mathrm{Au}} \Delta T + m_{\mathrm{Cu}} C_{\mathrm{Cu}} \Delta T \\ &= \big(m_{\mathrm{Au}} C_{\mathrm{Au}} + m_{\mathrm{Cu}} C_{\mathrm{Cu}}\big) \Delta T \\ &= \big[m_{\mathrm{Au}} C_{\mathrm{Au}} + (100 - m_{\mathrm{Au}}) C_{\mathrm{Cu}}\big] \Delta T \\ &= \big[m_{\mathrm{Au}}(0.0301) + (100 - m_{\mathrm{Au}})(0.0923)\big](23.4)\,. \end{aligned} \tag{5}$$

For the next step, we can write

$$\frac{200}{23.4} - (100)(0.0923) = m_{\mathrm{Au}}[(0.0301) - (0.0923)]\,. \tag{6}$$

For the last step, we get

$$m_{\mathrm{Au}} = \frac{8.547 - 9.23}{0.0301 - 0.0923} = 10.98\,[\mathrm{g}]\,, \tag{7}$$

So, this sample is about 11% gold and 89% copper. Therefore, the claim is false.