Specific Heats Problem 2

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Abstract

Here we treat a problem in specific heat as a mere algebra word problem.

1 Problem Statement

A 100 g piece of ice at -40 °C is added to 100 g of water at 40 °C. What will be the final equlibrium configuration of the system? (Assume that heat is distributed only between the ice and water.)

2 Setup

Let Q_x be the heat added to a substance of mass m_x and specific heat C_x. The *specific heat* of a substance is the amount of heat energy needed to raise the temperature of the substance 1° C. The equation that relates these thermodynamic variables is

$$Q_x = m_x C_x \Delta T\,, \tag{1}$$

where T is the temperature in kelvin or °C. Either scale will work because ΔT deals only in temperature differences.

So, for ice and water, we have that

$$Q_{\text{Ice}} = m_{\text{Ice}} C_{\text{Ice}} \Delta T \quad \text{and} \quad Q_{\text{Water}} = m_{\text{Water}} C_{\text{Water}} \Delta T\,, \tag{2}$$

where

$$C_{\text{Ice}} = 0.5\,\text{cal/g}\cdot{}^\circ\text{C} \quad \text{and} \quad C_{\text{Water}} = 1.0\,\text{cal/g}\cdot{}^\circ\text{C}\,, \tag{3}$$

but we also need to know the *Latent heat of fusion* (L_{f}) for ice, which is

$$L_{\text{f}} = 80\,\text{cal}\,/\,\text{g}\,. \tag{4}$$

3 Scheme & Solution

So, where to start? For myself, I'd like to go to cases! What *could* happen?

(a) All of the ice will melt.
(b) None of the ice will melt.
(c) Only some of the ice will melt.

Note 1: The order I placed these items in is not nearly so important as the fact that they collectively give us a starting point to analyze this problem.

Note 2: At thermal equalibrium the mix of water and ice must be at one temperature. This means that if all the ice is gone that temperature will be at freezing temperature or above. And, if there is only ice left, that temperature will likely be at freezing temperature or below. And if there is both water and ice, they will be at freezing.

Case (a). Will all of the ice melt?

Before we can melt the ice, we have to bring it up to 0°C.

How much heat energy will that require?

$$Q_{\text{Ice}} = (100\,\text{g})(0.5\,\text{cal/g}\cdot{}^\circ\text{C})(40\,{}^\circ\text{C}) = 2000\,\text{cal}\,. \tag{5}$$

How much heat is available in the water at 40 °C if we take it down to 0 °C?

$$Q_{\text{Water}} = (100\,\text{g})(1\,\text{cal/g}\cdot{}^\circ\text{C})(40\,{}^\circ\text{C}) = 4000\,\text{cal}\,. \tag{6}$$

This is more than enough to bring the ice to 0 °C, but it's still in solid form. We have 2000 calories left over, but is that enough to melt all the ice?

How much heat do we need to add to the ice at 0 °C to melt it?

$$Q_{\text{Ice to melt}} = (80\,\text{cal}\,/\,\text{g})(100\,\text{g}) = 8000\,\text{cal}\,. \tag{7}$$

But we only have enough heat left over from cooling the warm water to melt one-fourth of the ice or 25 grams. Therefore the final equlibrium configuration of the system is 75 grams of ice at 0 °C, and 125 grams of water at 0 °C.

So, it turns out that Case (c) was correct.