Teaching Stoichiometry as Algebraic Word Problems, Part 2

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Abstract

Stoichiometry has been called the mathematics of general chemistry. In Part 1 of this series, we saw how that the approach to solving algebra word problems called *Scheme*, originally developed to solve challenging algebra word problems, can be adapted easily to solving stoichiometry problems. This second paper continues in the same line.

1 Introduction

Stoichiometry is a basic topic of chemistry, concerned with solving for certain quantities of products and/or reactants in (usually) a balanced chemical equation, given knowledge of other quantities of products and/or reactants in the equation. Such quantities of interests are typically moles, grams, and/or liters of particular substances.

In Appendix A, you'll find a compilation of molar masses of common chemicals. In Appendices B, C, and D you'll find a number of word problems related to chemistry, but not normally considered stoichiometry, per se. In Appendix E, you'll find a brief overview of my own mathematical mystery tour that led me from math/science to computer science and back to math/science, with the cognitive advantages of using 1) top-down problem solving with stepwise refinements and 2) flowcharting math theorems, to reveal their hidden patterns.

In Part 1 of this series, we saw how that the *Scheme* stoich diagram manifests the origin of the stoichiometric proportion as a scaling relationship between the MoleStats line and the Moles line. In most treatments I've seen of stoichiometry, this mole proportion is obscured,[1] being skipped and replaced by a series of multiplications by conversion factors. But when one encounters a stoichiometry problem as difficult as found below in Problem 3 (A Two-Step Dehydration Process), one is forced to solve it using old-fashioned algebra in multiple unknowns (or prove me wrong).

[1]A proportion is a literal equation of the form $a/b = c/d$.

A final controversy before beginning. What do we mean by a *chemical reaction* in the first place? One reply is that a chemical reaction occurs when new substances are produced from old substances. But how to define a 'new substance'? Another reply is that a chemical reaction occurs when initial chemical bonds are broken and new bonds created. Let's test these definitions: Let's dissolve x grams of salt in y liters of pure water. The 'chemical equation' for this is

$$\mathrm{NaCl(s)} \longrightarrow \mathrm{Na^{+}(aq) + Cl^{-}(aq)}\,. \tag{1}$$

Is this a true chemical reaction? Some say No, because no new substance was created. Others say Yes, because bonds were broken and reframed differently.

2 Problem 1: Finding Volume of Evolved Gas not at STP

This next problem is taken from the YouTube chemistry course presented by Tyler DeWitt.[2]

PROBLEM: If 85.3 g of NaN_3 decomposes at 75°C and 2.30 atm, what volume of N_2 will be made? (Ans: 24.5 L)

SOLUTION: Step 1.

We begin with a balanced equation.

$$\mathrm{2NaN_3(s)} \longrightarrow \mathrm{2Na(s) + 3N_2(g)}\,. \tag{2}$$

Now, we will solve the problem for the volume V using the Ideal Gas Law $PV = nRT$ to solve for the volume in terms of the pressure ($P = 2.30$ atm), the temperature ($T = 348$ K), and the moles (n) of the N_2 gas. To help us calculate n, we'll employ a stoich diagram:

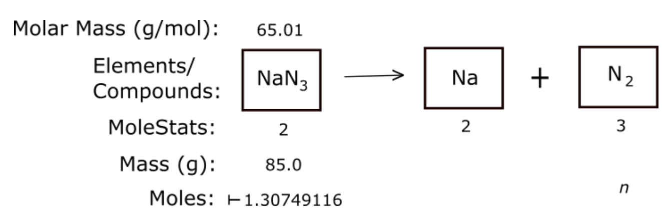

Figure 1. A sparse diagram, displaying only data relevant to solving this problem. N_2 is treated as an ideal gas with mole count n.

We need a word about the notation in the above figure. There are four main types of data in the stoich diagrams I make. The most common are data from given information, from the coefficients of the balanced equation, and from data

[2]See https://www.youtube.com/watch?v=qvOVXg24Npo .

tables, such as a periodic table of elements for molar mass information. This kind of data I do not mark up. The second kind of data in stoich diagrams comes from computations based on data in the same column, for which I use the turnstile ($\vdash$) to indicate. The third kind of data is a result in one column that required data from at least one other column to calculate it, and I indicate that kind of value or result by use of the underlining. The fourth kind of data in the figures is the result of combining given information to derive a secondary value. I indicate this kind of data with a right arrowhead (▶).

Next, we write down our mole proportion from columns 3 and 1:

$$\frac{3}{2} = \frac{\text{moles N}_2}{\text{moles NaN}_3} = \frac{n}{1.30749116\,\text{mol}}\,. \tag{3}$$

Solving for n, we get

$$n = 1.96123674\,\text{mol}\,. \tag{4}$$

Step 2.

Solving the Ideal Gas Law for V, we get

$$V = \frac{nRT}{P}\,. \tag{5}$$

Using $R = 8.2057 \times 10^{-2}\,\frac{\text{atm}\cdot\text{L}}{\text{mol}\cdot\text{K}}$, we have

$$V = \frac{(1.96123674\,\text{mol})(8.2057 \times 10^{-2}\,\frac{\text{atm}\cdot\text{L}}{\text{mol}\cdot\text{K}})(348\,\text{K})}{2.30\,\text{atm}} = 24.3\,\text{L}. \tag{6}$$

3 Problem 2: How Long Will the Lithium Hydroxide Last?

This next problem is taken from the online chemistry site:

`https://talk.collegeconfidential.com/ap-tests-preparation/233538-hard-stoichiometry-problem.html`

PROBLEM:

The space shuttle environmental control system handles carbon dioxide (4% by mass exhaled air) by reacting it with Lithium Hydroxide pellets to form lithium carbonate and water. If there are 7 astronauts on board the shuttle, and each exhales 20 liters of air per minute, how long could clean air be generated if there were 25,000 g of lithium hydroxide pellets available for each shuttle mission? Assume the density of air is 0.0010 g/mL. (Proposed Ans: 4109 minutes.)

SOLUTION: Step 1.

We begin with a balanced equation.

$$2\text{LiOH} + \text{CO}_2 \longrightarrow \text{Li}_2\text{CO}_3 + \text{H}_2\text{O}\,. \tag{7}$$

Question: How is this even a stoichiometry problem in the first place? Answer: Because the rate of consumption of the LiOH (which we'll assume to be a constant) is related to the rate of production of the CO_2 (which we'll also assume to be a constant).

Let's make a stoich diagram to help out.

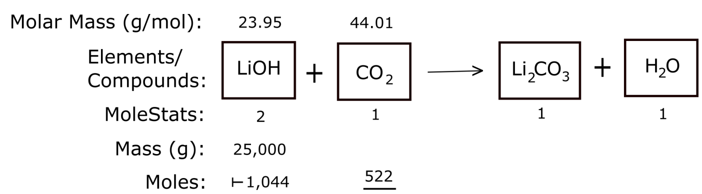

Figure 2. Here we used the underlining of data, which means that its determination required data from at least one other column.

Now, we apply the Zeroth Rule of Problem Solving and assume that the CO_2 is being consumed at the same rate it's being produced.

Next, we write down the mole proportion between columns 1 and 2:

$$\frac{2}{1} = \frac{\text{moles LiOH consumed}}{\text{moles CO}_2\text{ consumed}}\,. \tag{8}$$

Now, if we let ΔT be the time until the LiOH is consumed, then (8) can be changed to

$$\frac{2}{1} = \frac{(\text{moles LiOH consumed})/\Delta T}{(\text{moles CO}_2\text{ consumed})/\Delta T}\,. \tag{9}$$

But the numerator of (9) represents the average rate of consumption of LiOH, and the denominator represents the average rate of consumption of CO_2. But we have assumed that the rate of consumption of CO_2 is equal to its rate of production, hence:

$$\frac{2}{1} = \frac{(\text{moles LiOH consumed})/\Delta T}{(\text{moles CO}_2\text{ produced})/\Delta T}\,. \tag{10}$$

Therefore, LiOH is consumed twice as fast as CO_2 is produced.

Step 2.

The fundamental question we need to answer is this: What is the duration of time ΔT (in minutes) that the LiOH will last, given the rate of production of CO_2? We can write this down as a simple equation:[3]

[3] This equation follows a very common form. Let $Q(T)$ be some quantity being produced or consumed at a fixed time rate of change R, then

$$\Delta T = \frac{Q}{R}\,. \tag{11}$$

$$\Delta T = \frac{\text{moles LiOH}}{\text{rate LiOH is consumed } [\text{mol} \cdot \text{min}^{-1}]} \,. \tag{12}$$

Next, let's calculate the rate of production of CO_2 in $g \cdot min^{-1}$ per person, $R^p_{CO_2}$:

$$\begin{aligned} R^p_{CO_2} &= (20\,\text{L} \cdot \text{min}^{-1}/\text{person})(0.04\frac{\text{g CO}_2}{\text{g air exhaled}})(0.0010\,\text{g/mL})(1000\,\text{mL/L}) \\ &= 0.8\,\text{g} \cdot \text{min}^{-1}/\text{person}\,. \end{aligned} \tag{13}$$

Therefore, the rate of production for all seven persons, R_{CO_2}, is

$$R_{CO_2} = 5.6\,\text{g} \cdot \text{min}^{-1}\,. \tag{14}$$

Next, we convert this into a rate of production in moles per minute $\overline{R}_{CO_2}$, using the molar mass of CO_2:

$$\overline{R}_{CO_2} = \frac{5.6\,\text{g} \cdot \text{min}^{-1}}{44.01\,\text{g} \cdot \text{mol}^{-1}} = 0.1272\,\text{mol} \cdot \text{min}^{-1}\,. \tag{15}$$

But, the molar rate of consumption of LiOH is twice the molar rate of production of CO_2:

$$\overline{R}_{\text{LiOH}} = 0.2544\,\text{mol} \cdot \text{min}^{-1}\,. \tag{16}$$

On substituting the calculated values into (12), we get

$$\Delta T = \frac{1044\,\text{mol}}{0.2544\,\text{mol} \cdot \text{min}^{-1}} = 4104\,\text{minutes}\,. \tag{17}$$

4 Problem 3: A Two-Step Dehydration Process

This next problem is taken from the Yahoo Answer site:

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https://answers.yahoo.com/question/index?qid
                          =20081230083424AA2lfMV
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PROBLEM:

Hard stoichiometry problem?

Can some show me how to do this?

You are given a mixture of three hydrated salts:

$$Na_2CO_3 \cdot 10H_2O, \quad MgSO_4 \cdot 7H_2O, \quad \text{and} \quad CuSO_4 \cdot 5H_2O\,.$$

The total mass of the mixture is 12.123 grams. When the mixture is heated gently, the following two reactions occur:

$$Na_2CO_3 \cdot 10H_2O(s) \longrightarrow Na_2CO_3 \cdot 7H_2O(s) + 3H_2O(g) \tag{18a}$$

$$MgSO_4 \cdot 7H_2O(s) \longrightarrow MgSO_4 \cdot H_2O(s) + 6H_2O(g) \tag{18b}$$

After these reactions are complete, the mass of the mixture has decreased to 9.049 grams. This mixture is then heated more strongly, and the following additional reactions occur:

$$Na_2CO_3 \cdot 7H_2O(s) \longrightarrow Na_2CO_3(s) + 7H_2O(g) \tag{19a}$$

$$MgSO_4 \cdot H_2O(s) \longrightarrow MgSO_4(s) + H_2O(g) \tag{19b}$$

$$CuSO_4 \cdot 5H_2O(s) \longrightarrow CuSO_4(s) + 5H_2O(g) \tag{19c}$$

After this final heating, the mass of the mixture has decreased to 6.412 grams. From this information, calculate the masses of each of the three compounds in the original mixture.

The provided answers are:

mass of $Na_2CO_3 \cdot 10H_2O$	1.374 g
mass of $MgSO_4 \cdot 7H_2O$	6.418 g
mass of $CuSO_4 \cdot 5H_2O$	4.331 g

SOLUTION: Step 1.

We first note that all the chemical equations displayed above are balanced. Next, we consider the paired equations in the first reaction, given by (18a) and (18b).

Let's set the initial values of moles of $Na_2CO_3 \cdot 10H_2O$, $MgSO_4 \cdot 7H_2O$, and $CuSO_4 \cdot 5H_2O$ to a, b, c, respectively. Once we know their values, we can calculate the masses of their respective hydrates quite easily.

Before we make stoich diagrams to help out, we should first make some simplifying notations for the three hydrates. For each generic hydrate Hy, the symbol Hy_n represents $Hy \cdot n\,H_2O$, and Hy_0 represents the anhydrous salt. Further, letting $X \sim Na_2CO_3$, $Y \sim MgSO_4$, and $Z \sim CuSO_4$, then, for examples

$$X_{10} \sim Na_2CO_3 \cdot 10H_2O,$$

$$Y_1 \sim MgSO_4 \cdot H_2O,$$

$$Z_0 \sim CuSO_4.$$

Now for a diagram depicting the first heating event and its consequences:

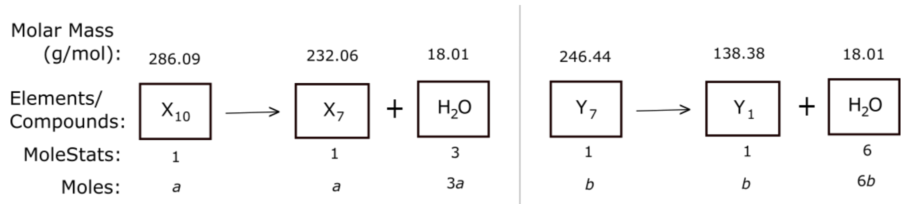

Figure 3a. Diagram of the two independent reactions. Here we've introduced the variables a and b to represent, respectively, the moles of X_{10} and Y_7. Once we know their values we can easily calculate their corresponding masses.

In Figure 3a, adopting the simplifying notations above, we represent the first two independent reactions that occurred in the first heating event. From the given information, we can calculate the mass of the evolved water gas. Second, the mole counts for the products X_7 and Y_1 will be passed along to the initial reactant mole counts during the second heating event.[4]

One thing we know for sure from the first reaction is the mass of the evolved water gas, being the difference between the original mass of the mixture and the residue mass of the mixture after the first heating event:

$$\text{Mass H}_2\text{O evolved in first reaction} = 12.123\,\text{g} - 9.049\,\text{g} = 3.074\,\text{g}\,, \tag{20}$$

which we can then write in term of its moles and the molar mass of water, $18.01\,\text{g}{\cdot}\text{mol}^{-1}$:

$$(3a + 6b)\text{mol}\,(18.01\,\text{g}{\cdot}\text{mol}^{-1}) = 3.074\,\text{g}\,, \tag{21}$$

where we used the fact that the total moles of water produced is the sum of the moles from the left reaction (being $3a$) and the moles from the right reaction (being $6b$). We can rewrite this last equation for moles more simply as

$$3a + 6b = 0.17068\,. \tag{22}$$

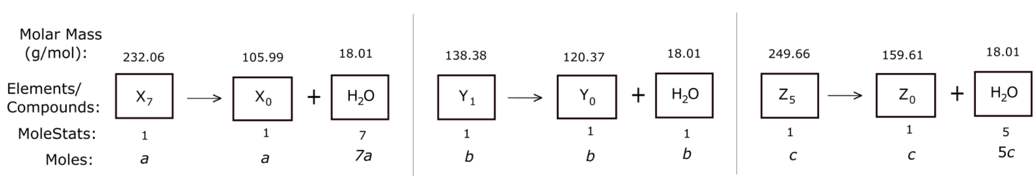

Figure 3b. This figure represents the second heating event.

Noting that Eq. (22) provides us with one equation to solve for the triple variables a, b, c, we need only two more equations, derived from the information

[4]An alternative graphic is found on page 33.

in Figure 3b, to form a set of three equations in three unknowns, which we will then solve simultaneously.

First, we know the mass of the final anhydrous salts to be 6.412 grams. Thus (Total is the sum of its parts)

$$\text{Mass of } X_0 + \text{Mass of } Y_0 + \text{Mass of } Z_0 = 6.412\,, \tag{23}$$

which can be expressed in terms of their moles and molar masses as

$$105.99a + 120.37b + 159.61c = 6.412\,. \tag{24}$$

And, arguing similarly to how we derived Eq. (22) for the mass of the water gas evolved in the first heating event, we have for the second heating event

$$(7a + b + 5c)\text{mol}\,(18.01\,\text{g}\cdot\text{mol}^{-1}) = 2.637\,\text{g}\,, \tag{25}$$

which simplifies to

$$7a + b + 5c = 0.146419\,. \tag{26}$$

So, on solving (22), (24), and (26) simultaneously for a, b, c, Wolframalpha gives

moles of $Na_2CO_3 \cdot 10H_2O$	$a = 0.00481197$
moles of $MgSO_4 \cdot 7H_2O$	$b = 0.0260407$
moles of $CuSO_4 \cdot 5H_2O$	$c = 0.0173389$

Then, converting these mole values to gram values, we get

mass of $Na_2CO_3 \cdot 10H_2O$	1.377 g
mass of $MgSO_4 \cdot 7H_2O$	6.417 g
mass of $CuSO_4 \cdot 5H_2O$	4.329 g

5 Problem 4: Production of Nitric Acid

This problem is taken from the online stoichiometry practice test:

`https://www.alvinisd.net/cms/lib03/TX01001897/Centricity/Domain/4240/practice%20test%20stoich.pdf`

PROBLEM:

#7. How many grams of nitric acid, HNO_3, can be prepared from the reaction of 138 g of NO_2 with 54.0 g H_2O according to the equation below?

$$3NO_2 + H_2O \longrightarrow 2HNO_3 + NO \tag{27}$$

a. 92   b. 108   c. 126   d. 189   e. 279 .

SOLUTION: Step 1.

We'll begin by treating this as a limiting reactant type problem.

Let's make a stoich diagram to help out.

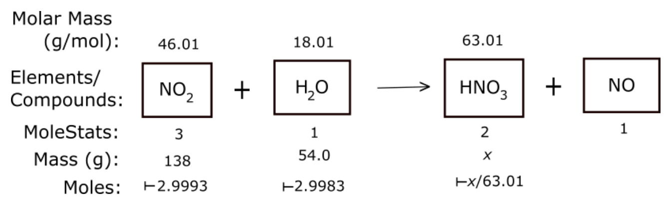

Figure 4. The problem told us that we have 54.0 grams of H_2O to use. Let's see if that's enough or too much.

Our first task is to decide which of the reactants is limiting. Say, we consume all the H_2O in the reaction. How much NO_2 would we need? From the Molestats line we see that for each mole of H_2O used, we need 3 moles of NO_2, but this obviously won't work. So, the NO_2 is the limiting reactant.

Therefore we assume that we use all the NO_2 and form the mole proportion for columns 3 and 1:

$$\frac{2}{3} = \frac{\text{moles HNO}_3}{\text{moles NO}_2} = \frac{x/63.01}{2.9993}\,. \tag{28}$$

Solving for x, we get about 126 grams, which is answer c.

6 Problem 5: A Gravimetric Analysis Problem

This problem is taken from the online YouTube second practice problem:

`https://www.youtube.com/watch?v=e7MOJiJZSiQ`

PROBLEM:

A 10.500 gram mixture contains calcium nitrate and potassium chloride. Excess lead (II) nitrate solution, $Pb(NO_3)_2$ (aq), is added to precipitate out 4.227 grams of lead (II) chloride, $PbCl_2$ (s). What percent by mass of potassium chloride is in the mixture?

SOLUTION: Step 1.

We'll begin with the equation that deals with the precipitation:

$$2\text{KCl(aq)} + \text{Pb(NO}_3)_2\text{(aq)} \longrightarrow \text{PbCl}_2\text{(s)} + 2\text{KNO}_3\text{(aq)}\,. \tag{29}$$

Let's make a stoich diagram to help out.

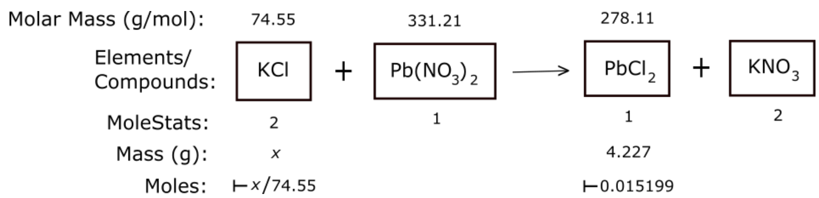

Figure 5. Lead (II) chloride ($PbCl_2$) is one of the few insoluble chlorides.

Next, we write down our mole proportion for the two compounds of interest from columns 1 and 3:

$$\frac{2}{1} = \frac{\text{moles KCl}}{\text{moles PbCl}_2} = \frac{x/74.55\,\text{g}\cdot\text{mol}^{-1}}{0.015199\,\text{mol}}\,. \tag{30}$$

Which gives for x about 2.226 grams. Thus, the percent by mass of the KCl in the original mixture is

$$\frac{2.226}{10.500} \times 100\% = 21.6\% \tag{31}$$

Answer given: 21.6%.

7 Problem 6: Gravimetric Analysis Problem 2

This problem is taken from the online practice problem:

`https://www.calstatela.edu/sites/default/files/dept/chem/07winter/201-lec/201-l-4-gravimetric-analysis.pdf`

Definitions:

FW = Formula weight = molar mass
ppt = precipitate

PROBLEM (p. 7):

Consider a 1.0000 g sample containing 75% potassium sulfate (FW 174.25) and 25% MSO_4. The sample is dissolved and the sulfate is precipitated as $BaSO_4$ (FW 233.39). If the $BaSO_4$ ppt weighs 1.4900, what is the atomic weight of M^{2+} in MSO_4?

Answer given: Mg^{2+}. (There's a problem here between what is given as the answer and what was asked for. I'll change the request to, Find the actual element that M stands for. This turns out to be Mg.)

SOLUTION: Step 1.

We clearly have yet another before-and-after process, but what should we identify as the main invariant of this process? To answer that question, we need a clearer idea of the process itself. First, we dissolve all the analyte in water. Second, we *apparently* add excess of barium ion Ba^{2+} until all the sulfate ions ${SO_4}^{2-}$ in solution have precipitated out in the form of $BaSO_4$.

We begin with an unbalanced equation[5] that deals with the precipitation:

$$Ba^{2+} + K_2SO_4(aq) + MSO_4(aq) \longrightarrow BaSO_4(s) + \text{other stuff} \qquad (32)$$

Step 2. Make a stoich diagram to help out.

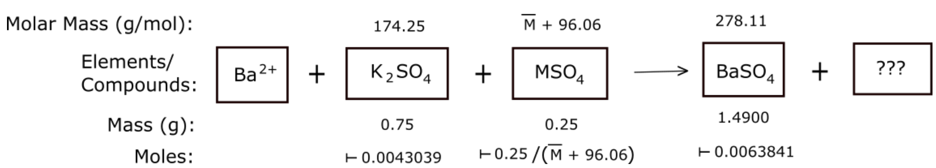

Figure 6. We need only track what happens to the sulfate ions, which are contained in columns 2, 3, and 4. The bar over the M means to take the molar mass of the element. The molar mass of the sulfate ion is 96.06 g/mol.

Step 3.

The masses of the two compounds in the analyte are easily calculated from the percentages each has of the original dry analyte (see the figure). Next, from the claim that *we add excess of barium ion* Ba^{2+} *until all the sulfate ions* ${SO_4}^{2-}$ *in solution have precipitated out in the form of* $BaSO_4$, we derive the conservation equation that the molar amount of ${SO_4}^{2-}$ ions in solution at the beginning is equal to the molar amount of ${SO_4}^{2-}$ 'ions' in the $BaSO_4$ precipitate; hence we can count them by moles in this before-and-after process:

$$\begin{aligned}(\text{moles } {SO_4}^{2-} \text{ from col. \#2}) + (\text{moles } {SO_4}^{2-} \text{ from col. \#3}) \\ = (\text{moles } {SO_4}^{2-} \text{ from col. \#4})\,. \qquad (33)\end{aligned}$$

Now, because there is a one-to-one ratio of moles of K_2SO_4 to moles of ${SO_4}^{2-}$ produced by dissolving, and a one-to-one ratio of moles of MSO_4 to moles of ${SO_4}^{2-}$ produced by dissolving, and, finally, a one-to-one ratio of moles of $BaSO_4$ to moles of ${SO_4}^{2-}$ taken out of solution by precipitation, then (33) becomes[6]

$$0.0043039\,\text{mol} + \frac{0.25\,\text{g}}{(\overline{\text{M}} + 96.06)\,\text{g}\cdot\text{mol}^{-1}} = 0.0063841\,\text{mol} \qquad (34)$$

[5]We won't bother to balance this equation because we won't be requiring the use of a stoichiometric ratio, in which case we should ask if this kind of problem fits the definition of stoichiometry.

[6]Herein is perhaps where the 'stoichiometry' lies in this problem.

Solving this last equation for $\overline{\mathrm{M}}$, yields

$$\overline{\mathrm{M}} = 24.12\,\mathrm{g{\cdot}mol^{-1}}\,. \tag{35}$$

Therefore, we conclude that Mg is the correct element corresponding to M.

8 Problem 7: Gravimetric Analysis Problem 3

This problem is taken from the online practice problem:

`https://www.calstatela.edu/sites/default/files/dept/chem/07winter/201-lec/201-l-4-gravimetric-analysis.pdf`

PROBLEM (p. 8):[7]

A mixture of mercurous chloride (FW 472.09) and mercurous bromide (FW 560.99) weighs 2.00 g. The mixture is quantitatively reduced to mercury metal (At wt 200.59) which weighs 1.50 g. Calculate the quantities of mercurous chloride and mercurous bromide in the original mixture. (I modified the question to fit the answer given.) ANS: 0.5182 g

SOLUTION: Step 1.

We clearly have yet another before-and-after process, but what should we identify as the main invariant of this process? Obviously, it's the conservation of the moles of the mercury from beginning to end. We also have a 'total is the sum of its parts' equation in which we know the total mass of both mercurous compounds

$$(\text{grams } Hg_2Cl_2) + (\text{grams } Hg_2Br_2) = 2.00\,\mathrm{g}\,, \tag{36}$$

Let's set x = grams Hg_2Cl_2, then grams $Hg_2Br_2 = 2.00 - x$.

Step 2. Now, make a 'stoich' diagram to organize all the information. Be sure to label the rows precisely!

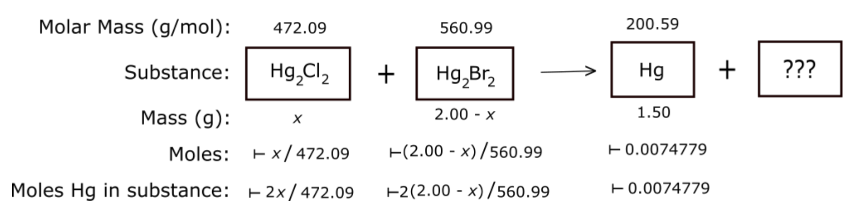

Figure 7. Our interest is the conservation of the moles of mercury. The conservation of mass of the two reactants is not of direct interest to us, but, rather, their total mass of 2.00 g provides a constituitive relationship between the masses of their respective mass contents.

[7]Definitions: FW = Formula weight = molar mass, 'At wt' = atomic weight.

Step 3.

We know that the moles of mercury is conserved:

$$(\text{moles Hg in } Hg_2Cl_2) + (\text{moles Hg in } Hg_2Br_2) = \text{moles Hg recovered}\,. \quad (37)$$

From the last line of Figure 7, we have that

$$\frac{2x}{472.09} + \frac{2(2.00 - x)}{560.99} = 0.0074779\,. \quad (38)$$

The reason for the 2's in the numerators of the fractions in the last equation result from the fact that for each mole of Hg_2X_2 we get 2 moles of Hg. Now, after using WolframAlpha to solve the last system of equations simultaneously, I got

$$x = 0.518\,\text{g}\,, \qquad 2.00 - x = 1.482\,\text{g}\,. \quad (39)$$

9 Problem 8: Gravimetric Analysis Problem 4

This problem is taken from the YouTube practice problem:

`https://www.youtube.com/watch?v=e7MOJiJZSiQ`

PROBLEM:

An unknown metal cation has a +1 charge (M^+). The bromide of the unknown metal, MBr, is dissolved in enough water to make 100.0 mL of solution. The solution is then mixed with an excess of $AgNO_3$ solution to precipitate AgBr (molar mass = 188 g/mol). The precipitate is collected by filtration, dried, and the following data was obtained:

Mass of MBr = 1.38 g

Mass of filter paper = 0.98 g

Mass of filter paper and AgBr = 2.86 g

What is the identity of the metal chloride?

a) KBr b) NiBr c) LiBr d) NaBr

Ans: b).

SOLUTION:

Step 1. Prepare data needed for the stoich diagram.

From the given information, we can derive the mass of the AgBr

$$\text{mass AgBr} = (\text{mass filter paper + AgBr}) - (\text{mass filter paper}) = 1.88\,\text{g}\,. \quad (40)$$

When this mass information is placed in the stoich diagram, it will be preceded by a ▶ symbol to inform the reader that the information was derived, not given or looked up in a reference source.

Step 2. Make a stoich diagram.

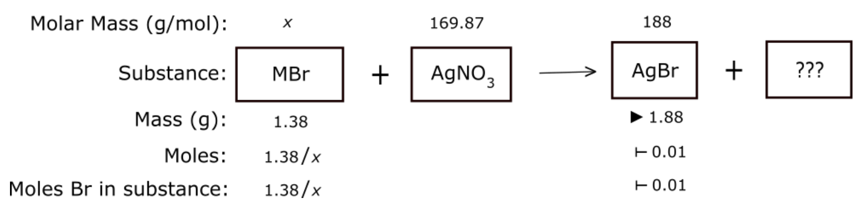

Figure 8. Once we know x, we can determine from a periodic table what M stands for. The turnstile means ‘is calculated from information only in the same column’. The solid right arrowhead means ‘is a derivative result of the given information or some other source’.

Step 3.

Now, equating the moles of Br in columns 1 and 3, and solving for x, we get at about $138\,\text{g}{\cdot}\text{mol}^{-1}$, of which $79.90\,\text{g}{\cdot}\text{mol}^{-1}$ is from the bromine. The difference, then, is approximately the molar mass of M, namely, $58.1\,\text{g}{\cdot}\text{mol}^{-1}$. Therefore, the mystery element must be nickel, making b) the answer.

10 Problem 9: A Combustion Problem

This problem is taken from the on-line PDF practice problem:

`https://www.quia.com/files/quia/users/lockarm/stoichiometry/activity---Stoichiometry-Word-Problems-2-SOLUTIONS.pdf`

PROBLEM #2:

Nitroglycerin, $C_3H_5(ONO_2)_3$, was invented in 1846 by an Italian chemist named Ascanio Sobrero. Nitroglycerin contains both an oxidant and a fuel. When it detonates, it decomposes to form carbon dioxide, water, nitrogen, and oxygen, all in a gaseous state. Every mole of the explosive that decomposes in this way generates a tremendous amount of energy – approximately 1.5 MJ (1 MJ = 1 megajoule = 1×10^6 J = 1 MJ).

a. If 1.135 kilograms of nitroglycerin detonates, how many total liters of gas (assuming STP) are produced?

Balanced Equation:

$$4C_3H_5(ONO_2)_3 \rightarrow 10H_2O + 12CO_2 + 6N_2 + O_2 \tag{41}$$

Ans: 812 liter.

b. How much energy is produced by the explosion?

Ans: 7.5 MJ.

SOLUTION:

Step 1.

Our first task is to calculate the volume of the products at STP. We will treat all the products as ideal gases, even the water because of the extreme heat generated. Now, we know that all ideal gases take up the same volume per mole, n, at the same temperature and pressure.

Step 2. Make a stoich diagram.

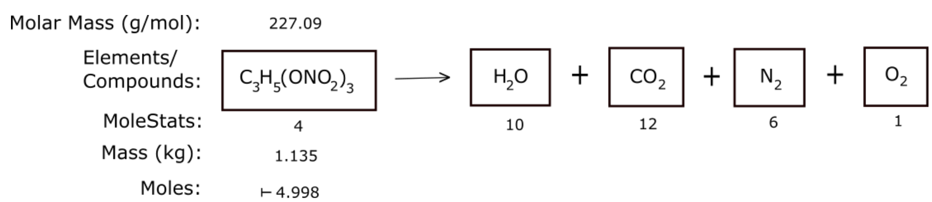

Figure 9. Four moles of nitroglycerin produces $10 + 12 + 6 + 1 = 29$ moles of an 'ideal gas mixture'. Part of problem solving is knowing when different things are the 'same' and when the 'same' things are different.

The stoichiometric proportion of interest to us from the figure is

$$\frac{4.998}{4} = \frac{n}{29}\,. \tag{42}$$

Solving this for n yields 36.24 moles. At STP a single mole of ideal gas takes up 22.4 liters. Therefore, the volume we seek is

$$V = n(22.4\,\mathrm{L{\cdot}mol^{-1}}) = 811.7\,\mathrm{liters}\,. \tag{43}$$

The answer to Part b is easy. Since 1 mole of the explosive produces 1.5 MJ of energy, 4.998 moles produces 7.497 MJ of energy.

11 Appendix A: Relative Molecular Masses

Atomic masses are given in terms of grams per mole ($g{\cdot}mol^{-1}$). For the compounds, I used the values given by
https://www.convertunits.com/

Ag — 107.87 (Silver)
AgBr — 187.77 (silver bromide)
AgCl — 143.32 (silver chloride)
Ag_2CrO_4 — 331.73 (silver chromate)
$AgNO_3$ — 169.87 (silver nitrate)

Al — 26.98 (Aluminum)
Al_2O_3 — 101.96
$Al(OH)_3$ — 78.00 (aluminum hydroxide)
AlC_3 — 133.34
$Al_2(CrO_4)_3$ — 401.94
$Al_2(SO_4)_3$ — 342.15

As — 74.92 (Arsenic)
As_4O_6 — 395.68

B — 10.81 (Boron)
B_2H_6 — 27.67
B_2O_3 — 69.62

Ba — 137.33 (Barium)
$BaCl_2$ — 208.23 (barium chloride)
$Ba(OH)_2$ — 171.34
$Ba(NO_3)_2$ — 545.80
$BaSO_4$ — 233.39 (barium sulfate)

Be — 9.01 (Beryllium)

Br — 79.90 (Bromine)
Br_2 — 159.81

C — 12.01 (Carbon)
CCl_4 — 153.82 (Carbon tetraflouride)
$CHCl_3$ — 119.38 (Chloroform)
CBr_2Cl_2 — 242.72
CH_3OH — 32.04

CH_3COOH — 60.05
CO — 28.01
CO_2 — 44.01
COC_2 — 98.92 (phosgene)
CH_2O — 30.03
CH_5NO_2 — 63.01 (ammonia formate)
C_2H_2 — 26.04 (acetylene)
C_2H_6 — 30.07 (ethane)
C_2H_4O — 44.53 (...)
C_3H_6O — 50.08
$C_3H_6O_3$ — 90.08 (lactic acid)
$C_3H_8O_3$ — 92.09
$C_6H_{12}O_6$ — 180.16
$C_6H_5CO_2K$ — 160.21 (potassium benzoate)
$C_3H_5(ONO_2)_3$ — 227.09 (nitroglycerin)
$C_7H_5(NO_2)_3$ — 227.13
CH_3 — 15.03 (methyl radical)
CH_4 — 16.04 (methane)
CH_3OH — 32.04
C_3H_8 — 44.10 (propane)
C_4H_8 — 56.11 (butene)
C_4H_{10} — 58.12 (butane)
C_5H_{10} — 70.13 (?)
C_5H_{12} — 72.15 (pentane)
C_8H_{18} — 114.23 (octane)

Ca — 40.08
$CaBr_2$ — 199.89
CaC_2 — 64.10
$CaCl_2$ — 110.98
$CaCl_2 \cdot 2\,(H_2O)$ — 128.99
CaO — 56.08 (calcium oxide)
$Ca(OH)_2$ — 74.09
$Ca_2(PO_3)_2$ — 270.10
$Ca_3(PO_3)_2$ — 310.18 (calcium phosphate)
$CaCO_3$ — 100.09
$CaSO_4$ — 136.14
$CaSiO_3$ — 116.16 (calcium metasilicate)

Cl — 35.45 (Chlorine)
Cl_2 — 70.91

Co — 58.93 (Cobalt)
$CoCl_2$ — 129.84 (cobalt chloride)

Cr — 52.00 (Chromium)
Cr_2O_3 — 152.00
$Cr(NO_3)_2$ — 176.01

Cs — 132.91 (Cesium)

Cu — 65.39
$CuCl_2$ — 134.45
$Cu(OH)_2$ — 97.56
$Cu(NO_3)_2$ — 183.56 (copper(II) nitrate)
Cu_2S — 159.16 (copper(I) sulfide)
Cu_2O — 143.09 (copper(I) oxide)
$CuSO_4$ — 159.61

F — 19.00
F_2 — 38.00

Fe — 55.93 (Iron)
$FeCl_2$ — 126.75
$FeCl_3$ — 162.20
Fe_2O_3 — 159.69 (iron(III) oxide)
$FeSO_4$ – 151.91
$Fe_2(SO_4)_3$ – 399.88
FeS – 87.91 (iron(II) sulfide)
$FeTiO_3$ – 151.71 (iron(II) titanate)

Ga — 69.73
Ga_2O_3 — 187.44 (gallium(III) oxide)

H — 1.01
H_2 — 2.02
HBO_2 — 43.82
HBr — 80.91 (hydrobromic acid)
$H_2C_2O_4$ — 90.03
$H_2C_4H_4O_6$ — 150.087
HCN — 27.06
H_3BO_2 — 45.83
HCl — 36.46
$HClO_4$ — 100.56 (perchloric acid)
HF — 20.01

HI — 127.91 (hydrogen iodide)
H_2O — 18.01
H_2O_2 — 34.01
HNO_3 — 63.01
H_3PO_4 — 24.31
H_2S — 34.08
H_2SO_4 — 98.08
H_2SO_3 — 82.01

Hf — 178.49 (Hafnium)

Hg — 200.59 (Mercury)
Hg_2Br_2 — 560.99 (mercurous bromide)
Hg_2Cl_2 — 472.09 (mercurous chloride)

I — 126.90 (Iodine)
I_2O_5 — 333.81 (diiodine pentoxide)

K — 39.10
KCl — 74.55
K_2CrO_4 — 194.19
$K_2Cr_2O_7$ — 294.18
KCN — 65.21
$K_4Fe(CN)_6$ —v368.34
K_2HPO_4 — 174.18
KIO_3 — 214.00 (potassium iodate)
K_3PO_4 — 212.27
KO_2 — 71.10
KOH — 56.10
$KMnO_4$ — 158.03
KNO_2 — 85.10 (potassium nitrite)
KNO_3 — 101.10 (potassium nitrate)
K_2SO_3 — 158.26 (potassium sulfite)
K_2SO_4 — 174.26 (potassium sulfate)

Li — 6.94
LiBr — 86.85 (lithium bromide)
LiCl — 42.39 (lithium chloride)
$LiClO_4$ — 106.39 (lithium perchlorate)
Li_2CO_3 — 73.89
L_3N — 34.83
$LiNO_3$ — 68.95

LiOH — 23.95 (lithium hydroxide)
Lil_2SO_4 — 109.94

Mg — 24.31
MgCl — 59.76
$MgCl_2$ — 95.21
MgF — 43.30 (magnesium flouride)
$MgCO_3$ — 83.31 (magnesium carbonate)
Mg_3N_2 — 100.93 (magnesium nitride)
MgO — 40.30 (magnesium oxide)
$Mg(OH)_2$ — 58.32
$MgSO_4$ — 120.37

Mn — 54.94 (manganese)
MnO_2 — 86.94
$Mn(NO_3)_3$ —240.95 (manganese (III) nitrate)
Mn_2S_3 —206.07 (manganese (III) sulfide)

N — 14.01
N_2 — 28.01
N_2l_2 — 30.03
NH_3 — 17.03
NH_4 — 18.01
$(NH_4)_2Cr_2O_7$ — 252.06
$(NH_4)_2CO_3$ — 96.09 (ammonium carbonate)
$(NH_4)Cl$ — 53.49
$(NH_4)ClO_4$ — 117.49
NH_4OH — 35.05
NH_4NO_3 — 80.04
NO — 30.01
NO_2 — 46.01
N_2O_5 — 108.01

Na — 23.00
NaBr — 102.89 (sodium bromide)
NaCl — 58.44 (sodium chloride)
$NaClO_4$ — 58.44 (sodium perchlorate)
NaCN — 49.01 (sodium cyanide)
Na_2CO_3 —105.99 (sodium carbonate)
$Na_2C_2O_4$ — 134.00
Na_2CrO_4 — 161.97
$Na_3C_6H_5O_7$ — 258.07
NaF — 41.99 (sodium flouride)

Na_3PO_4 — 163.94
$NaHCO_3$ — 84.01
$NaIO_3$ — 197.89 (sodium iodate)
NaN_3 — 65.01
$NaNO_3$ — 84.99
$NaKC_4H_4O_6$ — 210.16
NaOH — 40.00
Na_2SO_4 — 142.04
$Na_2S_2O_3$ — 158.11

Ne —20.18

O —16.00
O_2 —32.00
O_3 —48.00

P — 30.97
P_4 — 123.90
P_4H_{10} — 133.97 (phosphorus pentoxide)
PH_3 — 34.00 (phosphine)
PH_4I — 161.91
P_2I_4 — 569.57

Pb —207.20
$PbCl_2$ — 278.11 (lead(II) chloride)
$PbCrO_4$ — 323.19
PbS — 239.27
PbO — 223.20
$Pb(SO_4)_2$ — 399.33
$Pb(NO_3)_2$ — 331.21 (lead(II) nitrate)
$Pb(NO_3)_4$ — 455.22

Ra —226.03 (Radium)

Rb — 84.87

S — 32.07
SO_2 — 64.06
$SO_4{}^{2-}$ — 96.06

Sb — 121.76 (Antinomy)

Sb_2O_3 — 291.52

Sc — 44.96 (Scandium)

Si — 28.09
SiO_2 — 60.08

Sr — 87.62 (strontium)
SrO — 103.62 (strontium oxide)

Ti — 47.88
$TiCl_4$ — 198.68 (titanium (IV) chloride)
Ti_2O_2— 127.73

U — 238.03
UF_6 — 352.02
U_3O_8 — 842.08

Y — 88.91 (Yttrium)

Zr —91.22

Zn — 65.39
$ZnCl_2$ — 136.29
$Zn(NO_3)_2$ — 189.39

Appendix B: Seven Chemistry-Related Word Problems Solved

All these problems are taken from the website

`https://www.algebra.com/algebra/homework/word/mixtures/Mixture-problems.lesson`

The general heuristics for solving all these problems will be as follows:

1) Make a diagram.
2) Account for conserved quantities.
3) Solve system of equations to get the answer.

Note: These algebra problems ignore issues of significant figures.

Problem 1: How much water should be added to 200 milliliters of a 10% salt solution to get a 2% salt solution?

(Concentrations here are mass-to-volume concentrations, measured in g/mL units, grams of the salt per 1 milliliter of the solution volume).

Solution to Problem 1

Let's make a diagram to help out.

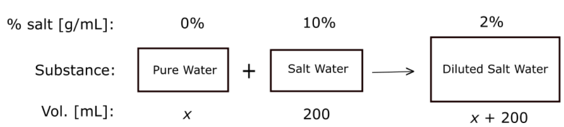

Figure B1. Pure water is added to salt water to dilute it.

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1) Conservation of volume is already accounted for in the diagram.
2) Next, we write down the conservation of salt equation:

$$(0.0)(x) + (0.10)200 = (0.02)(x + 200)\,. \tag{44}$$

The solution for x is 800 mL.

Problem 2: How much water must be evaporated from 1000 milliliters of a 2% salt solution to get a 10% salt solution?

(Concentrations here are mass-to-volume concentrations, measured in g/mL units, same as the last problem.

Solution to Problem 2 Make a diagram to help out.

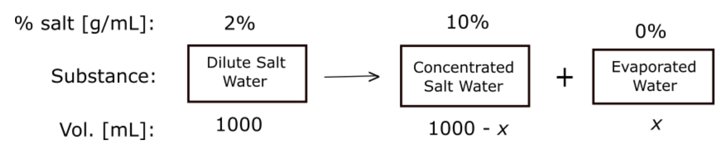

Figure B2. Water is evaporated from salt water to concentrate it.

1) Conservation of volume is already accounted for in the diagram.
2) Next, we write down the mass conservation of salt equation:

$$(0.02)(1000) = (0.10)(1000 - x) + (0.0)x\,. \tag{45}$$

The solution for x is 800 mL.

Problem 3: How much salt should be added to 1000 milliliters of a 2% salt solution to get a 4% salt solution?

(Concentrations here are mass-to-volume concentrations, measured in [g/mL] units, same as before. [Assume that the volume of water is not changed by adding this salt.]

Solution to Problem 3:
Let's make a diagram to help out.

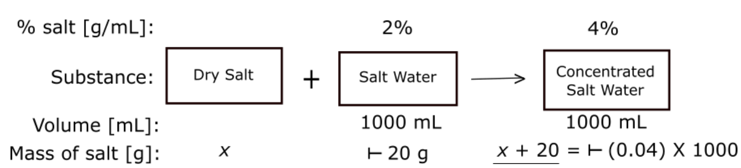

Figure B3. The 2% salt solution has $(0.02\ \text{g} \cdot \text{mL}^{-1})(1000\ \text{mL}) = 20$ g of salt.

1) Conservation of volume is already accounted for.
2) Next, we write down the 'mass conservation of salt' equation:

$$x + 20\,\text{g} = (0.04\,\text{g} \cdot \text{mL}^{-1})(1000\,\text{mL}) = 40\,\text{g}\,. \tag{46}$$

The algebraic solution for x is 20 g.

Comment:

Perhaps the turnstile and underlining look a bit confusing at first, but after the student is familiar with them, they should make the stoich diagrams much easier to understand. I refer to the class of these markups as *Co-ops* which is short for *Cognitive Operators*, which are unary operators intended to improve understanding.

When I invented *Scheme* to solve algebra word problems,[8] the origin of the data in the *Scheme* diagrams was pretty much obvious. But when I adapted *Scheme* for stoichiometry, I soon noticed that the origin of data in the stoich diagrams was much more difficult to grasp at a glance. This is primarily because in stoichiometry, either one gathers data from many different sources to work a problem, or one manipulates the given data prior to placing it in the diagram. So, to aid the reader in understanding the origin of data in a stoich diagram (in a particular row and column), and hence quickly grasping its meaning, I invented or co-opted unary operators (turnstile, underlining, etc) to give added meaning to the data in the diagrams.

Problem 4:
How much water should be added to 200 milliliters of a 10% acid solution to get a 2% acid solution? (Concentrations here are volume-to-volume concentrations, measured in [mL/mL] units, or milliliters of the acid per 1 milliliter of the solution).

Solution to Problem 4 Let's make a diagram to help out.

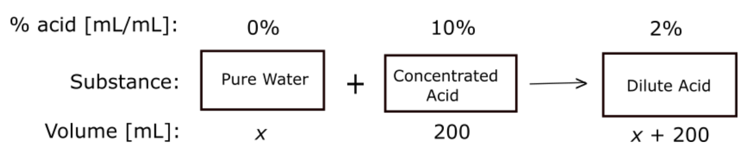

Figure B4. Here, the notion of 'concentrated' vs 'dilute' acid is not following any technical definition. To illustrate what I mean, sulfuric acid is said to be 'concentrated' only when at or above 98.3% by mass.

1) Conservation of volume is accounted for.
2) Next, we write down the conservation of 'pure' acid equation:

$$(0.00)(x) + (0.10)(200) = (0.02)(x + 200). \tag{47}$$

The solution for x is 800 mL.

Problem 5:
How much of the pure acid should be added to 1000 milliliters of a 2% acid solution to get a 4% acid solution? (Concentrations here are volume-to-volume concentrations, measured in [mL/mL] units, same as in Problem 4).

[8]See Appendix C in the previous Stoichiometry paper in this series to see how this was done.

Solution to Problem 5 Let's make a diagram to help out.

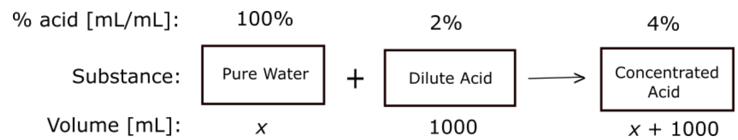

Figure B5. The terms 'dilute' and 'concentrated' are not standard, just convenient to use as labels.

1) Conservation of volume is already accounted for.
2) Next, we write down the conservation of 'pure' acid equation:

$$(1.00)(x) + (0.02)(1000) = (0.04)(x + 1000). \tag{48}$$

The solution for x is 20.8 mL.

Problem 6:
How much of a 10% acid solution should be added to 1000 milliliters of a 2% acid solution to get a 4% acid solution? Concentrations here are volume-to-volume, part-to-whole, measured in [mL/mL] × 100% units.

Solution to Problem 6:
Let's make a diagram to help out.

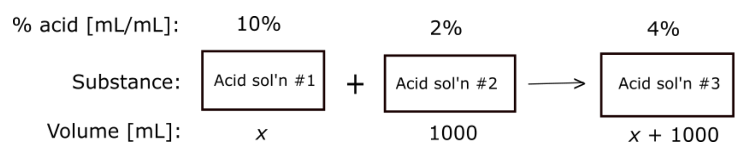

Figure B6. Apparently, the point of so many similar problems is for the student to grasp this kind of problem from every angle.

1) Conservation of volume is accounted for.
2) Next, we write down the conservation of 'pure' acid equation:

$$(0.01)(x) + (0.02)(1000) = (0.04)(x + 1000)\,. \tag{49}$$

The solution for x is 333.3 mL.

Problem 7:
How much pure antifreeze liquid should be added to 1 gallon of 40% antifreeze to

get 60% antifreeze? (Concentrations here are volume concentrations, measured in [volume/volume] units).

Solution to Problem 7:
Let's make a diagram.

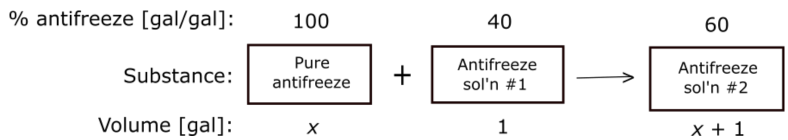

Figure B7. How unmathematical when I pour some arbitrary amount of antifreeze into my car's radiator. Algebra! Who knew?

1) Conservation of volume is already accounted for.
2) Next, we write down the conservation of 'pure' antifreeze:

$$(1.00)(x) + (0.4)(1) = (0.6)(x + 1)\,. \tag{50}$$

The solution for x is 0.5 gal.

Appendix C: Three More Chemistry-Related Word Problems

These problems are taken from the website

`https://www.algebra.com/algebra/homework/word/mixtures/`
`Advanced-mixture-problems.lesson`

Problem 1 from the webpage: A chemist has three different acid solutions.[9] The first acid solution contains 15% acid, the second contains 30% acid, and the third contains 75% acid. He wants to use all three solutions to obtain a mixture

[9]When an author of an algebra problem says that there are 'three different acid solutions' to be added together, I interpret it to mean that there are three different concentrations of the same acid (chemically speaking) to be combined. One must be on the alert when solving a percentage problem a math teacher made up that involves 'chemistry'. Most of the time they work just fine. But, occasionally, they don't make sense from the get go.

of 216 liter containing 25% acid, using 2 times as much of the 75% solution as the 30% solution. How many liters of each solution should be used?

Solution to Problem 1:
Let's make a diagram to help out.

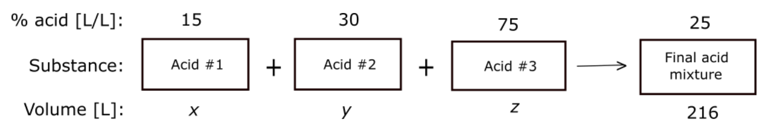

Figure C1. My preference is to treat this problem as a 'three-variable' problem, and not employ accelerated substitution by using the given constituitive relations between the variables in the diagram as volume descriptions.

As it stands, we have three unknowns, so we'd better find three equations to solve for them. From the requirement that the third volume be twice as much as the second, we have the constituitive equation[10] given by

$$z = 2y\,. \tag{51}$$

Now we investigate the two conservation equations. First, for volume:

$$x + y + z = 216\,, \tag{52}$$

and, second, for the conservation of 'pure' acid:

$$(0.15)x + (0.30)y + (0.75)z = (0.25)(216). \tag{53}$$

The solution for this system is

$$x = 168\,\text{gal}, \quad y = 16\,\text{gal}, \quad z = 32\,\text{gal}\,. \tag{54}$$

Problem 5 from the webpage: A certain concrete mixture contains 5.00% cement and 7.00% sand. How many kilograms of this mixture and how many kilograms of sand should be combined with 255 kg of cement to make a batch that is 16.0% cement and 18.0% sand?

Solution to Problem 5:
Let's make a diagram to help out.

[10] A *constituitive relation* is a relation/constraint existing on one or more unknowns (variables) that is (usually) arbitrary constructed and exists independent of the conservation equations.

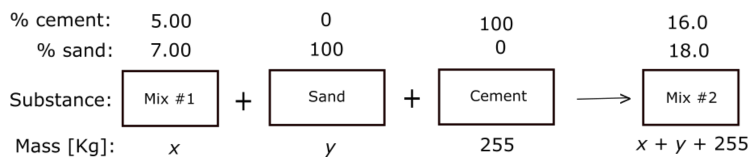

Figure C2. Conservation of mass is already accounted for.

Our first equation comes from the conservation of cement:

$$0.05x + 0.0y + (1.00)(255) = (0.16)(x + y + 255)\,. \tag{55a}$$

Our second equation comes from the conservation of sand:

$$0.07x + 1.00y + (0.0)(255) = (0.18)(x + y + 255)\,. \tag{55b}$$

The solution is

$$x = 1561\,\text{Kg}, \tag{56a}$$
$$y = 265\,\text{Kg}\,. \tag{56b}$$

Problem 6 from the webpage: When weighed in water, tin loses 0.137 of its weight and copper loses 0.112 of its weight. If an alloy of tin and copper weighing 18 pounds loses 2.316 pounds when weighed in water, how many pounds of each are there in the piece [of alloy]?[11]

Solution to Problem 6:

Let's make a diagram to help out.

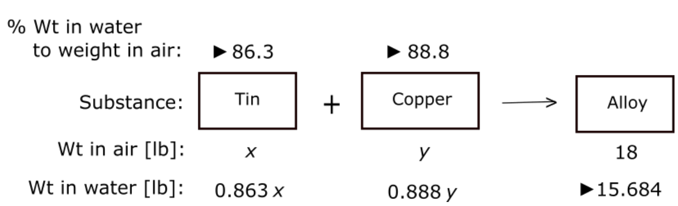

Figure C3. We imagine that the alloy is constructed from its parts.

I will be using some derived quantities in the diagram for this problem, so I should derive them first. So first we have to interpret what the author means

[11]By 'weight' we mean the number we get when using a weighing device. So, the weight of an object can change due to its environment, though the object's mass is invariant. We account for the difference in weights as being due to buoyancy forces of the water on the objects.

by 'tin loses 0.137 of its weight' in water. I interpret it to mean that tin weighs only 86.3% of its weight in air when weighed in water. Likewise, copper weighs only 88.8% of its weight in air when weighed in water. Second, 18 lbs of copper when weighed in air weighs only $(18 - 2.316)$ lbs = 15.648 lbs under water.

By conservation of weight in air, we get:

$$x + y = 18\,. \tag{57}$$

By conservation of weight in water, we get:

$$0.863x + 0.888y = 15.68\,. \tag{58}$$

The solution is for x and y is:

$$x = 12.16\,\text{lbs},$$
$$y = 5.84\,\text{lbs}\,.$$

Appendix D: An Alloy Problem

This problem is found at the website

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https://www.algebra.com/algebra/homework/word/mixtures
        /Mixture_Word_Problems.faq.question.954102.html
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Problem (paraphrased):

Starting with 100 lbs alloy of 20% copper and 5% tin, how many pounds of copper and pounds of tin must be melted into it to produce a new alloy that's 30% copper and 10% tin?

Solution:

The essence of a before-and-after process is that, while some things are clearly changing, other things presumably are not. In this problem, the things that are not changing are the total amounts of tin and copper, from which we derive two coupled conservation equations.

We will represent the amount of tin to be added as x and the amount of copper to be added as y. In Figure D1, we can see that our requirement that the weights of the constituent parts is preserved.

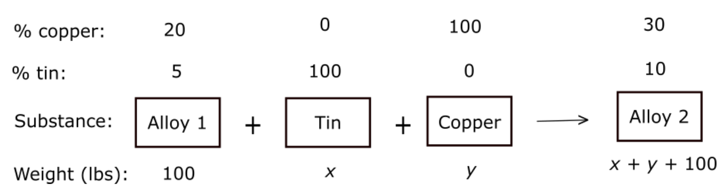

Figure D1. This graphic represents adding three things together, instead of the usual two things. Reasonably, pure tin has no copper in it, and pure copper has no tin in it. Conservation of weight is already accounted for.

We have two unknowns to solve for, so we need two coupled equations to solve for them.

$$\text{Conservation of Copper:} \qquad .2(100) + 0(x) + 1.0(y) = .3(x + y + 100)\,, \quad (59a)$$

$$\text{Conservation of Tin:} \qquad .05(100) + 1.0(x) + 0(y) = .1(x + y + 100)\,. \quad (59b)$$

My solutions are $x \approx 7.5$ and $y \approx 17.5$.

Appendix E: A Couple Memes Denied

Overview

A *meme*[12] is a notion that eventually gets spread through a population by its perceived merit. There are two notions of cognitive significance in computer science, in particular, that I want to emphasize that are popular by virtue of their effectiveness in problem solving, namely, the *top-down design with stepwise refinement*[13] and the use of flowcharting. These are basic tools of every computer science student, at least when I took the introductory classes decades ago.

Unfortunately, these cognitive notions in computer science have shown themselves difficult to cross the Computer Science boundary into related disciplines, especially into mathematics.

Flowcharting

Let's begin with flowcharting. Sure, every science uses flowcharts these days. But that's not my point. My point is that flowcharting is not yet rigorously taught to students as a general purpose, everyday technique of problem solving in science and mathematics. For examples: Why aren't geometry students required to present their geometry proofs in flowchart form? Why aren't chemistry students required to demonstrate in a flowchart the procedure for a titration experiment? These are algorithmic uses of flowcharts, and fairly straightforward to understand.

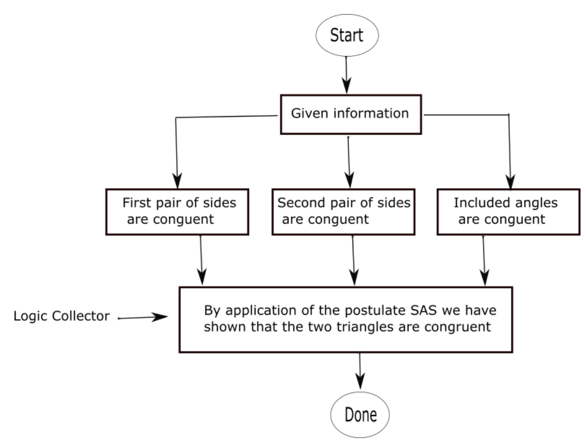

Figure E1. This graphic reveals the non-linearity of some geometry proofs by laying bare the parallelism in the subproofs involved.

For a case in point, consider a typical homework problem in high school geometry: If one is to use the SAS postulate to prove that two triangles are

[12]This term was introduced by Richard Dawkins in 1976.
[13]I presented this meme in some detail in the last paper of this series.

congruent (reference Figure E1), say, one has three subproofs to prove, i.e., that three corresponding parts are congruent. Generally speaking, each of these subproofs is independent of the others and a flowchart can make that obvious by diagramming them in parallel, not in series. But in a paragraph proof or in a two-column proof, this parallelism is obscured.

Okay, so we can use flowcharts for simple homework 'showthat' type problems, but what else? Of course, we can use math algorithms for repeated use, taking in various inputs and yielding various outputs, such as an algorithm to find the roots of an arbitrary quadratic in one variable. To see what people have done in algebra flowcharting, try entering the phrase 'algebra in a flowchart' into a search engine and look at Images.

Of the two memes, the one for flowcharting has fared a bit better in math and science, but not nearly as well as I image that it should go. Of course, flowcharts have been used for decades in math/science to represent algorithms and complex procedures. One of the earliest flowcharts I can remember, I saw in a 1970s chemistry textbook diagramming a complex analyte precipitate process. But what I find lacking is the encouragement of students to use flowcharts in their **everyday** problem solving.

For example, consider the following **alternative** flowchart that could be made to help solve Problem 3 on page 5:

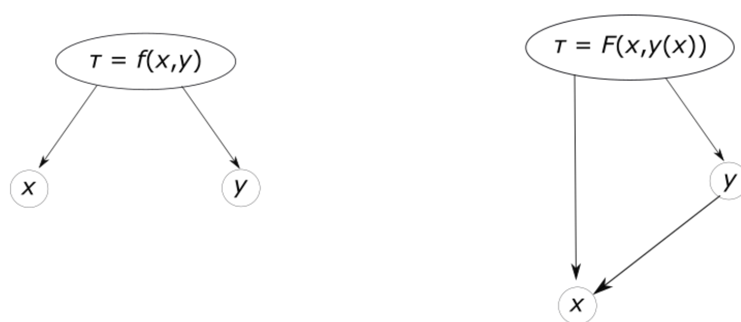

Figure E2. This graphic reveals a molar flowchart structure for Problem 3, where the starting points are the ovals and the end points are the rectangles.

So, what's the big deal over Figure E2? Well, by it we can visualize the 'Big Picture' of this complex collection of chemical reactions occurring over this double-heating event. The inputs and outputs are clearly marked. But, more importantly, the chart shows how the two heating events are coupled in such a way as to facilitate a stoichiometric analysis.

Top-Down Design

Now to the other meme of Top-Down Design with Stepwise Refinements: I went over this in some detail in the last paper of this series, so I won't belabor that here. But I do wish to repeat just enough of it to get the main idea across. Say our word problem to solve is to find the time it would take two printers, working at different rates, R_1 and R_2, to complete a specific print job.

Scheme has taught us to search for totals and parts. Okay, there is a total of **one job** being done by two contributing printers: Printer 1 and Printer 2. We know that every total is **equal** to the sum of its parts. So, let's introduce the shorthand 'part of job done by' → PJDB. Then our highest-level equation is

$$1\text{ job} = (\text{PJDB Printer 1}) + (\text{PJDB Printer 2})\,. \tag{60}$$

Then, by stepwise refinements, this last equation can be whittled down to standard algebra in the form

$$1 = R_1T_1 + R_2T_2\,. \tag{61}$$

My personal Journey into all this

I took my first computer class at university in the early 1980s (while finishing my undergraduate degree in mathematics), in which I learned of both memes. By 1985, I was adapting flowcharting to solve high school geometry problems (for tutoring purposes). Today, this graphical technique has found very limited usage in high school mathematics. Figure E3 reveals my personal timeline of utilizing these two memes in novel ways (at least to me) in my own math/science studies.

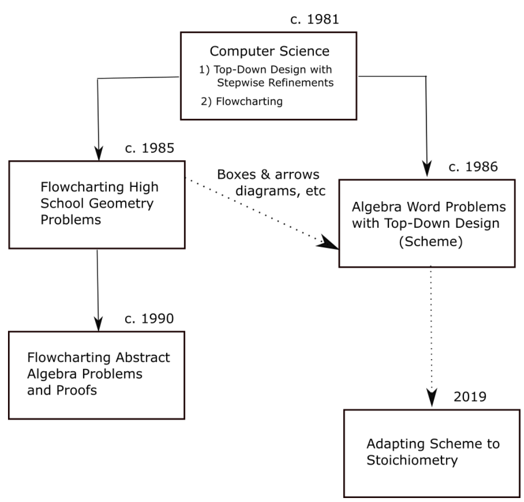

Figure E3. This graphic reveals the timeline of two memes (top-down design, flowcharting) that I carried from computer science into my math/science studies.

Yes, there's a lot of good flowcharting for high school geometry (for the few that use it); however, the use of flowcharting for secondary mathematics and advanced mathematics has scarcely moved beyond that. Just enter 'flowchart geometry' or 'math theorems in a flowchart' into a search engine to see what other people have done with it.

More Flowcharting: Real Proofs

In a theorem flowchart, one sees the typical necessity, in all but the easiest problems, to employ decision statements (nodes, usually diamond shaped) in the diagram. I refer to these nodes as 'logic splitters'. These statements are of the tautological form $P \vee \neg P$ (read 'P or not P'), where P is a logical proposition (that is, it's either true of false). Because such a statement is always true, it can be inserted anywhere without messing up the logic, but it remains for the flowchart developer to demonstrate its usefulness in a given problem.

Placing logic splitters into a flowchart of an established algorithm where it arises naturally is a no-brainer. For an example of where a logic splitter would arise naturally in a given algorithm, consider the situation where one is determining the roots of a quadratic equation in one variable and D is the discriminant, as in Figure E4.

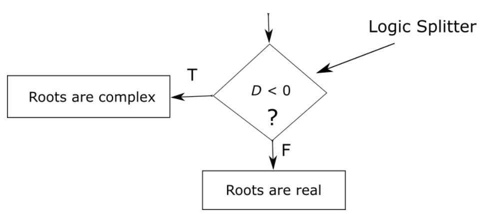

Figure E4. Obviously, either the discriminant D is negative or it's not.

Now, for an example of a theorem in a flowchart where the logic splitter is not obvious. Let's do a flowchart proof of the Fundamental Theorem of Arithmetic by the second form of mathematical induction (I won't include a proof of the uniqueness part of the theorem).

This theorem states that every natural number n greater than or equal to 2 is factorable into the form

$$n = \prod_{i=1}^{k} p_i \,, \tag{62}$$

where each p_i is a prime and the factors need not be distinct. To anticipate the limiting case of the 'factorization' of n when n is itself a prime, we'll 'factor' it as $n = p_1$. We'll employ a proof by induction, with our inductive hypothesis that *every* natural number greater than 1 and less than n is factorable into a product of primes. The flowchart proof of the Fundamental Theorem of Arithmetic is in Figure E5. Some obvious stepwise justifications have been left to the reader to provide.

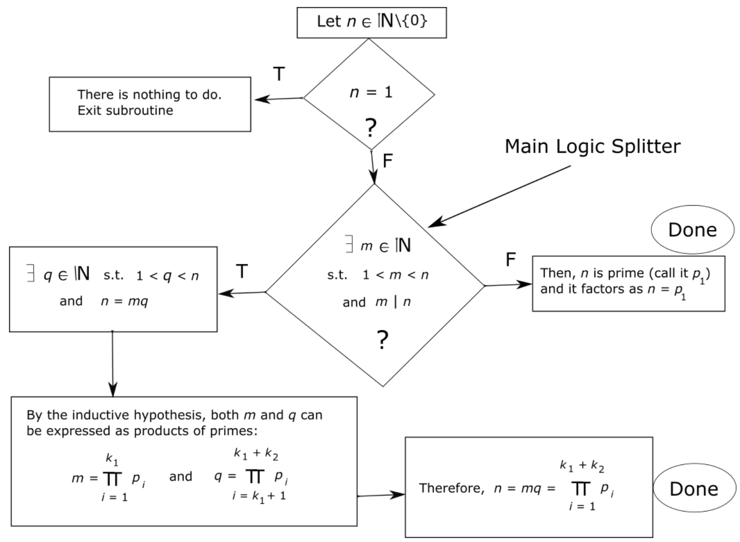

Figure E5. Flowchart for the Fundamental Theorem of Arithmetic [existence part], using the second form of mathematical induction. Note: $k = k_1 + k_2$.

By the way, I did not need to start the proof at step $n = 1$, but I included it for those who think that induction proofs require it.

Now that we have a real-life example to consider, let's make some observations on the benefits of flowcharting theorem proofs, starting with the logic splitter.[14]

The logic splitter in a theorem-proof flowchart plays the role in a proof that a plot device[15] plays in a story. Both story and proof have beginnings, middles, and ends, and both often need help in the middle to push things along.

It's been said that one advantage of doing a proof by contradiction is that in that approach one adds information to the system of givens to help find a useful conclusion (in that case, a contradiction). The logic splitter is similar, but much more general, for one can add in any proposition P so long as one tests both P and $\neg P$.

For example, say you're looking to prove the existence of an element a of set S that satisfies condition Q, i.e., that $Q(a)$ is true. Then set your proposition P to '$\forall\ x \in S$, $Q(x)$ is false'. The negation of a 'for all' is a 'there exists'. In this case, $\neg P$ implies '$\exists\, a \in S$ s.t. $Q(a)$ is true'. And, voila: we have a pro forma means to shoehorn into the logic flow of a proof a test for the existence of a particular element of a set!

Beyond the psychological barriers to the potential meme of flowcharting theorems, there are also practical reasons to resist them. Even if the idea of

[14] I go into a lot more detail in my paper *Group Theory and the Logic of Proof.*

[15] A *plot device* in a story helps move the plot forward, toward the end of the story.

flowcharting theorem proofs were to catch on, publishers would still be reluctant to employ them liberally. First, because they take up more space than an equivalent paragraph proof (spaghetti code[16]). Second, because they are labor intensive to construct. And third, because they can be a pain to find their best placement on a printed page. But, in spite of these concerns, I think they're well worth it.

This idea that a flowchart proof takes up too much space on the page is more than just considerations of the overall length of the book or journal article it resides in. There is this irrational feeling some people get called (in the publishing world) "Fear of the white space." There is a time in a presentation when less is better because it's easier for the mind to comprehend a smaller unit of information. We should resist the urge to fill up the white space of a flowchart with maximal information. Such diagrams can be hard to understand and are often referred to as being 'too busy'.

And one more reference to computer programming: Indentation of computer code is usually promoted by computer experts because it makes the code easier to read and to understand. Yet, indentation of code can generate a lot of white space on the page.

Okay, am I suggesting, for example, that Andrew Wiles's 129-page proof of Fermat's Last Theorem should be recast into flowchart form? Maybe so – sections of it, anyway. But one thing I know: I remember seeing Wiles presenting his explanation of the long process he used to find his proof, and he did so with the help of a flowchart on a chalkboard.

Historical Perspective on Flowcharting

I know what some of you must be thinking: Flowcharting has been around for a long time, so why blame this fictitious semipermeable boundary between Computer Science and Mathematics? Well, yes, flowcharting in one form or another has been around for decades. According to the website

`https://www.studymode.com/essays/History-Of-Flow-Chart-1846619.html`

> The first structured method for documenting process flow, the "flow process chart", was introduced by Frank Gilbreth to members of the American Society of Mechanical Engineers (ASME) in 1921 in the presentation "Process Charts — First Steps in Finding the One Best Way".

In fact, I'd say that flowcharting has been around for as long as engineers have made schematics of the flow of fluids in a complicated piping system. But I am interested in generalized flowcharts, where abstract things 'flow' through

[16] *Spaghetti code* is a term used in computer science to put down code that's hard to follow by its unvirtuous lack of 'structure' to reveal its logic flow.

the diagram, such as project goals, authority structures, logic. Thus, in PERT diagrams,[17] or work-flow diagrams, we deal with activities that have to be performed by prescribed dates, by assign persons responsible for the activities, and account for all contingencies along the way.

I can think of other kinds of flowcharts, such as those revealing 'subordinating' structure among members of a set or group, like ranks in the military or varieties of plants (kingdom, phylum,..., genus, specie), or authority relationships in a company. These examples seem to fall under the definition of *taxonomy*, which does sometimes use flowcharts, or something like it. For example, consider the taxonomy of $n \times n$ matrices found at

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https://networkscience.wordpress.com/2012/05/04/
    taxonomy-of-matrices/
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See Figure E6.

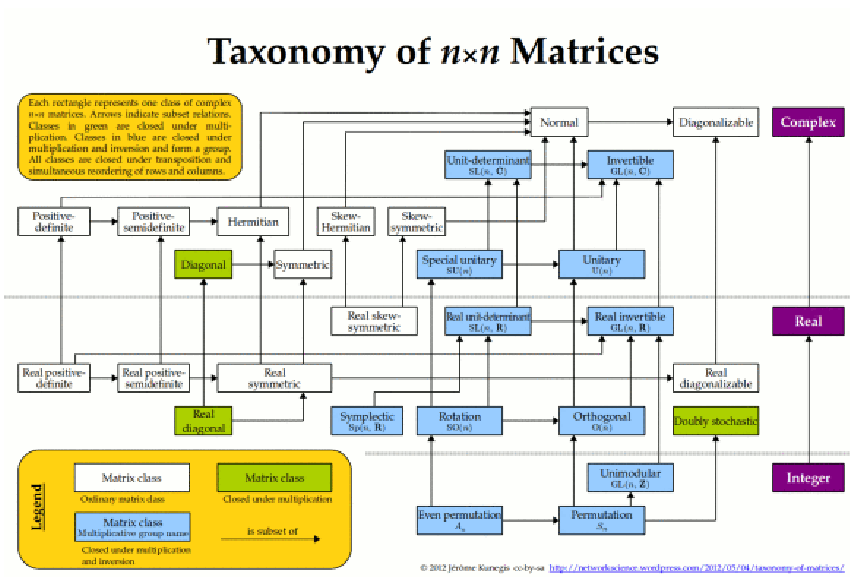

Figure E6. This graphic is used by permission under Creative Commons Licensing Agreement.

Quite recently, I saw a photograph of a 400 year old book opened to a particular page, on which was a genealogy (in the form of ovals and line segments) that filled up one whole page.

A Flowchart from Physics

For some time now, I have had on my bedroom wall a flowchart of the proof that one can use Maxwell's equations to prove that in a region of space free

[17]PERT stands for 'Program Evaluation Review Technique'.

of charges or currents that the **E** field behaves as a wave, which propagates at speed $1/\sqrt{\epsilon_0 \mu_0}$. I find such a flowchart, not only compact, but also beautiful.

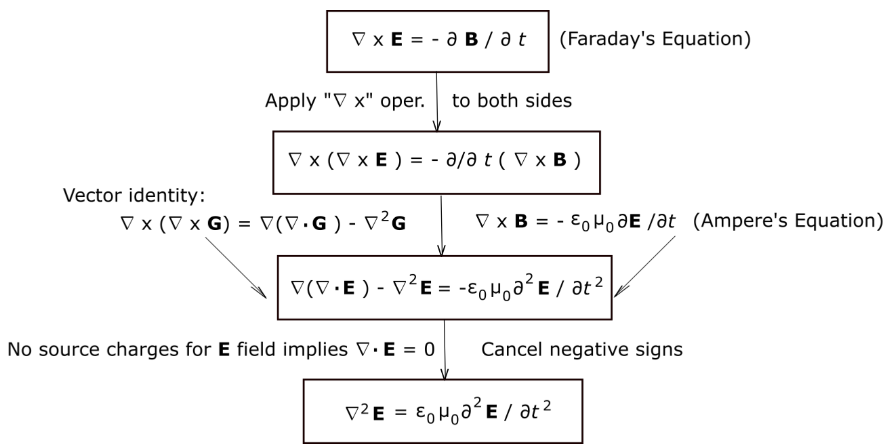

Figure E7. Here we have a flowchart proof that, in the absence of charges and currents, each component of the **E** field satifies a wave equation.

Although it's true that the flowchart in Figure E7 has a similarity to a two-column proof, I contend that my flowchart version is superior, mainly because the LHSs and RHSs of the equations can be both manipulated and annotated independently of each other, all without loss of clarity.

Definition of a Flowchart

So, what do we mean by a 'flowchart'? According to the website

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https://er.yuvayana.org/flowchart-history
              -definition-benefits-limitation/
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> Flow charts are easy-to-understand diagrammatic representation to showing how steps in a process fit together.

According to the Wikipedia

> A flowchart is a type of diagram that represents an algorithm, workflow or process. Flowchart can also be defined as a diagrammatic representation of an algorithm (step by step approach to solve a task).

Now, is a graphic a flowchart if it's not labeled as such? Well, yes, it can be. In fact, flowcharts are quite ubiquitous, hidden in plain sight. For example, a Feynman diagram is a flowchart. The 'arrow chasing' diagrams of category theory are flowcharts. And so on. Perhaps it all depends on how one defines a flowchart. Now it's my turn.

I define a *flowchart* as a nonempty collection of a finite number of nodes connected by line segments (edges), where a nonempty subset of these nodes are *starting nodes* and another nonempty distinct subset of the nodes are *finishing nodes*. Using the language of graph theory, such a structure is called a (generalized) *graph*. Every starting node must have at least one pathway to at least one finishing node. And every finishing node must have at least one inverse path to a starting node. The number of times a given node is traversed must be finite. These generalized graphs may have loops whereby nodes may be traversed more than once (e.g., loops in algorithms). They may also have loops from a given node to itself (e.g., identity morphisms on objects in category theory).

The line segments represent relationships between the pairs of nodes they connect. Typically, there is only one relationship per graph, but more relationships are allowed so long as the various relationships are visually distinguishable and consistently applied. Case in point: A typical genealogy flows from top to bottom. Following the arrows between two vertical nodes means 'whose offspring is', whereas, the horizontal relationships mean 'is the spouse of'. Some genealogies use a neutral horizontal bar as a logic collector, which serves to collect all offspring from a pair of spouses (i.e., their parents), from which line segments run down to the offspring.

Whether the line segments have arrows or not, there is a flow from the start nodes to the finish nodes. Therefore, these relationships are not all symmetric. Usually, all these relationships are of one type: 'the next step to be performed is', 'successor is', 'is followed by', 'logically implies', 'whose offspring is', 'to be accomplished by such-and-such a time', etc. Referring again to a top-down genealogy, vertical relationships are unsymmetric, whereas horizontal relationships are (or at least can be written as) symmetric.

Is the Notion of a Flowchart Old Fashioned or Outdated?

Well, some say that it is. After all, these days we have such 'modern' things as data-flow diagrams, action diagrams, sequence diagrams, etc. However, my short answer is, No, of course not. I have defined the flowchart so generally that all these are subsumed by the notion of flowchart. One definition to rule them all, you might say.

On the other hand, there are graphical representations of things and/or relationships that do *not* require start nodes or finish nodes, and they would not be flowcharts, but they're still very useful.

For example, what immediately comes to my mind is the logical equivalence of the Axiom of Choice, the Well-Ordering Principle, and Zorn's Lemma (Figure E8).

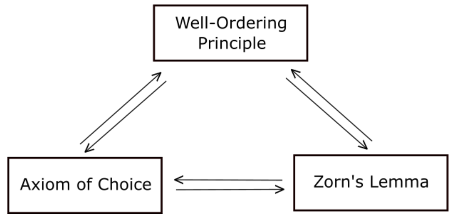

Figure E8. No starting or finishing nodes are needed in this diagram. The relationship assigned to each arrow is 'logically implies'. The relationship assigned to the double arrow is 'logically equivalent to'.

By the way, as a consequence of the logical equivalence of the three theorems, it follows that if one of them is true/false, then they're all true/false. But our human intuition probably fails us here. As the Wikipedia article on Zorn's Lemma puts it:

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"The Axiom of Choice is obviously true, the well-ordering principle
    obviously false, and who can tell about Zorn's lemma?"
                            -- Jerry L. Bona
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Greater Horizons

I trust that I have made my case that I'm not promoting flowcharting to the exclusion of all other forms of data visualization. Indeed, the subject of data visualization has become a vast discipline on its own, called *Infographics*, which Wikipedia defines as

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Infographics (a clipped compound of "information" and "graphics")
 are graphic visual representations of information, data or
 knowledge intended to present information quickly and clearly.
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Of recent importance in education is the *concept map*, which Wikipedia defines as

> A concept map or conceptual diagram is a diagram that depicts suggested relationships between concepts. It is a graphical tool that instructional designers, engineers, technical writers, and others use to organize and structure knowledge.

Most of the example images of concept maps I saw on the Web easily fit my definition of a flowchart. But I can think of other infographics that are not flowcharts, such as a graphic demonstrating the hydrologic cycle, which has no starting or finishing points.

So, I'm really very much in favor of expanding our horizons in the use of graphical aids, not arbitrarily restricting them. To that point, I'll make a final plea for people to expand their use of flowcharts, especially in problem solving.

Consider three problems. First, from computer science: Write a program to test the primality of a given integer. Second, from plumbing: Write a procedure to fix a leaking pipe under the sink. And third, prove Cauchy's theorem in group theory, that if p, a prime, divides the order of a finite group G, then G has an element of order p. What are the commonalities here?

So, what do we mean by "problem solving" in the first place? *Problem solving* is a rational process of going step-wise from a well-defined starting state to a well-defined finishing state. And that's exactly what flowcharts can easily model.

Conclusion

Although flowcharts have academic uses well beyond problem solving, my emphasis in this appendix has been to argue for a much more robust application of flowcharts to everyday problem solving in science and mathematics, for professionals and for students. Visualize and conquer.

References

[1] P. Atkins and L, Jones. *Chemical Principles: Quest for Insight*, 3rd Ed. Freeman (2005).

[2] M. Hein and S. Arena *Foundations of College Chemistry*, alternate 12th ed, John Wiley & Sons (2007), 421–422.

[3] M. S. Silberberg. *Chemistry: The Molecular Nature of Matter and Change* 4th Ed. McGraw-Hill (2006).