

# Spherical Geometry Problem from SciManDan

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## Abstract

In a recent YouTube video, creator SciManDan challenged Flat-Earthers to try their hands on an interesting spherical geometry problem. Now, I am not a Flat-Earther, but I decided to try to solve the problem myself.

## 1 Introduction

On 3 February 2024, SciManDan displayed the following figure, from which certain technical questions were posed as a challenge to Flat-Earthers, whom he claims that they claim to know more about the globe than the Globers (by knowing the heliocentric model of the solar system better than them).

<https://www.youtube.com/watch?v=utUurejAAv0>

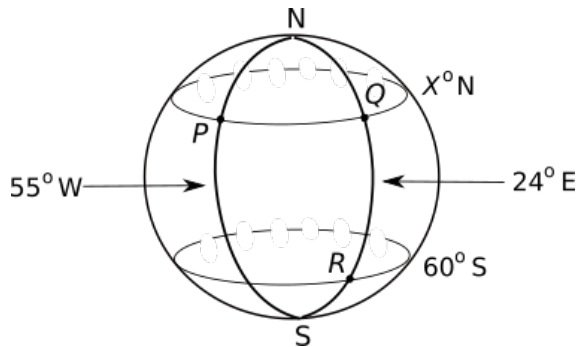


Figure 1. All questions are drawn from this figure (I don't know the source).

We are told that the figure is a representation of earth, and to take as the value of  $\pi$  as 3.142, and the value for the earth's radius as  $R = 6370$  km. Then we are given the three problems:

1. Calculate the difference in longitude between points  $P$  and  $R$ .
2. Calculate latitude  $X$ , given the values: the distance between points  $P$  and  $Q$ , which lie on latitude  $X$ , as 4392.079611 km.
3. Calculate the distance  $QR$  on longitude  $24^\circ$  E.

**Problem 1.**

Referencing Fig. 1, we see that the difference in longitude between points  $P$  and  $R$  is the same as between points  $P$  and  $Q$ , which is clearly  $|55^\circ - (-24^\circ)| = 79^\circ$ .

**Problem 2.**

I chose to solve this problem in a manner that I am most comfortable with, which is by setting up a standard coordinate system (used by physicists). Placing the center of the earth at the origin of coordinates, I have that

$$\begin{cases} x = R \sin \theta \cos \varphi, & (1a) \\ y = R \sin \theta \sin \varphi, & (1b) \\ z = R \cos \theta, & (1c) \end{cases}$$

where, as we are on a sphere,  $R$  has a fixed value, and where  $\theta$  is the polar angle from the  $z$ -axis (North Pole), and where  $\varphi$  is the azimuthal angle measured from the  $x$ -axis, going anticlockwise in the  $x, y$  plane. (See the figure below.)

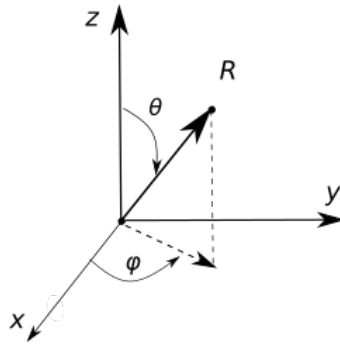


Figure 2. My preferred coordinate system to use. I will align the  $x$ -axis with  $55^\circ$  West with respect to the globe presented in Fig. 1.

On superimposing these last two figures, we get

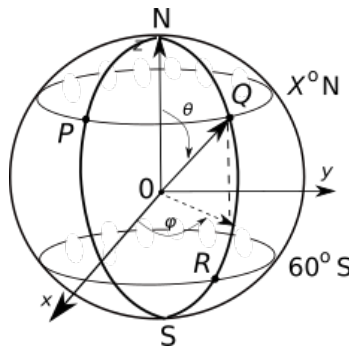


Figure 3. My preferred coordinate system superimposed in the original globe figure.

In Fig. 3, the  $x$ -axis is at  $\varphi = 0$  and the plane containing points  $N$ ,  $Q$ , and  $R$ , (i.e., the longitude at  $24^\circ$  E) is at  $\varphi = 79^\circ$ .

Now, latitude is measured in degrees above and below the equator, so I will have to convert to

that measuring standard when I find angle  $\theta$  from the North Pole, since I'll need the complement of that angle

$$X^\circ = 90^\circ - \theta. \quad (2)$$

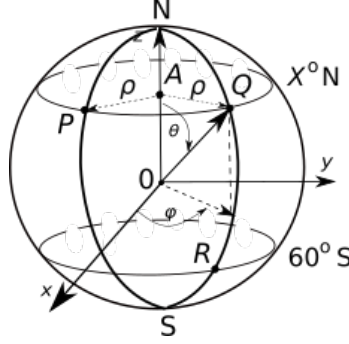


Figure 4. Added to the prior figure is the point  $A$ , which, together with points  $P$  and  $Q$ , form a plane parallel to the  $x, y$ -plane. And let  $\rho$  be the distance from point  $A$  to point  $P$ . (It's also the distance from  $A$  to  $Q$ .)

Continuing from the remarks in the last figure, the circle containing points  $P$  and  $Q$  (with center  $A$ ) has radius  $\rho$  given by the simple trigonometric relation

$$\rho = R \sin \theta. \quad (3)$$

So, we can then write

$$\sin \theta = \frac{\rho}{R}. \quad (4)$$

The circumference  $C$  of the circle through points  $P$  and  $Q$  is given by

$$C = 2\pi\rho. \quad (5)$$

Next, I want to introduce the symbol  $d$  for the arc length along the curve from  $P$  to  $Q$ .

$$d \equiv \text{dist}[\widehat{PQ}] = 4392.079611 \text{ km}. \quad (6)$$

Now it's time to employ a standard proportion in this kind of problem.

$$\frac{79^\circ}{360^\circ} = \frac{d}{C} = \frac{d}{2\pi\rho}. \quad (7)$$

Solving for  $\rho$ , we have that:

$$\rho = \frac{d}{2\pi} \frac{360}{79} = \frac{d}{2\pi} (4.557). \quad (8)$$

Using this and (4), we get

$$\begin{aligned} \theta &= \sin^{-1} \left[ \frac{\rho}{R} \right] \\ &= \sin^{-1} \left[ \frac{4392.079611}{6370} \frac{1}{6.284} (4.557) \right] \\ &\approx \sin^{-1}(0.500) \\ &= 30^\circ. \end{aligned} \quad (9)$$

Hence, from (2),

$$X^\circ = 60^\circ. \tag{10}$$

**Problem 3.**

For starters, the angle between  $X^\circ$  N and  $60^\circ$  S is, of course,  $120^\circ$ . Again, I want to introduce a symbol  $\delta$  to represent the arc length along the curve from  $Q$  to  $R$ , by

$$\delta \equiv \text{dist}[\widehat{QR}]. \tag{11}$$

And again, it's time to employ a proportion as we did above:

$$\frac{\delta}{2\pi R} = \frac{120^\circ}{360^\circ}. \tag{12}$$

Then

$$\begin{aligned} \delta &= \frac{1}{3}(2\pi R) \\ &= \frac{2}{3}(3.142)(6370\text{km}) \\ &= 13,343\text{km}. \end{aligned} \tag{13}$$

## 2 Conclusion

I had my mathematics education in the US many decades ago, and at no time did I ever see a problem such as this one, where one deals with latitudes and longitudes. Maybe these days it is taught in high schools. Anyway, I have seen spherical geometry in calculus and physics in college. That's why a) I chose the physics standard way of assigning angles, and b) I was unaware of the 'formula' that Scott used to aid in the solution to this problem.

However, there are only three theorems one needs to solve this problem from first principles: The first is that of finding the length of a side of a right triangle, given the hypotheuse and angle (usually one will use trigonometry). Second, the formula that relates the radius of a circle to its circumference. And three, that the arc length of a circle subtending  $\theta^\circ$  is to the circumference of the circle as the measure of the subtended angle  $\theta^\circ$  is to  $360^\circ$ . And, of course, one needs to know what a proportion is. Beyond that, one needs to know a bit of algebra and some arithmetic.