Heuristics, the Art of Problem Solving

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February 23, 2020

Abstract

This paper is a redo of an article that first appeared in the *Arizona Journal of Natural Philosophy*, April, 1991. The need for rigorous training in heuristics is examined. Certain techniques are recommended. The emphasis in this paper is on the kinds of heuristics geared toward technical problem solving.

1 Introduction

As an introduction, let me categorize heuristics into two broad camps, both using many common techniques. The first is what I'll refer to as *everyday heuristics*, such as defined on the psychologytoday.com webpage:

> A heuristic is a mental shortcut that allows an individual to make a decision, pass judgment, or solve a problem quickly and with minimal mental effort.

However, in this paper, I will stress *technical heuristics*, which I'll define as:

> *Technical heuristics* are the set of problem-solving rules of thumb or guidelines used to solve usually difficult technical problems, whose solutions require specialized knowledge and usually require a great deal of time and mental effort. They may concern problems that have resisted solution by many others for decades or even centuries. Or they may be useful for solving a student's homework problems that don't immediately reveal their solutions to the student.

I'll also define *general heuristics* as those problem-solving techniques common to both.

2 Technical Heuristics

There is perhaps no more important area of study to the mathematician, physicist, and engineer than that of *heuristics*, the theory and practice of problem solving. It's a shame that this subject is rather neglected in formal education.

Heuristics is an art form—a highly technical art form—but an art form just the same.

Traditional education in physics, for example, considers the student well educated if he or she does well in theory and experiment, but there is much more to contend with to be truly up-to-date with a robust education. Also needed is a foundation in the history and philosophy of physics, computer methods, ever more mathematical methods, and most importantly, heuristics. This paper will focus on those general problem-solving techniques of widest scope, and in future articles I'll present more specific heuristics in mathematics and physics.

Heuristics can be divided into two groups of techniques: those that produce algorithms and those that don't. Heuristics can also be divided into those techniques that produce known solutions—the usual case with textbook problems—and those that produce something truly creative, unknown to anyone else.

The first step in preparing to solve a problem is to believe that you can solve it. The second step is to recognize the task you must perform as a formal problem susceptible to formal problem-solving techniques. I offer the following rough definition of a problem. A *problem* is a task that you cannot immediately conceive of an algorithm to solve it. An *algorithm* is a method of performing a task by dividing the task into clear, unambiguous steps whose execution is completable in a finite amount of time. A recipe is a good example of an algorithm.

One last point before beginning. Every time anyone arrives at a solution to a problem that is new to him or her, then a certain amount of intuition has been used. Intuition is that part of us that allows us to believe in things that we can't prove.

Now I list my favorite heuristics, given in mostly random order. The exceptions are the first few, which I consider the most important. The first on the list I call *The Zeroth Rule of Problem Solving.*

0) Make any assumption necessary to solve the problem in a reasonable amount of time with a reasonable amount of effort.

1) "Keep an open mind—that's the secret." (Quote from *Doctor Who.*)

2) Be prepared to be a maverick. Everyone else may be doing something inefficiently or even incorrectly.

3) Even if you're a nobody, Nature doesn't mind: It reveals its secrets to nobodies who care. (Paraphrase from "The Galaxy Being," *The Outer Limits.*)

4) Neatness counts. Get organized and stay that way.

5) Invent your own algorithms and heuristics. Don't assume that those of your teacher, text, office or technical manual are best.

6) If the solution you obtain seems correct but doesn't jive with reality, check it again, redo it from the foundation up (the approach you used may contain false assumptions), question every piece of data (even a reference book can be

in error).

7) If you're stuck in the middle of a solution, explain your solution to someone. Changing your relation to the problem from solver to explainer could give you an insight. If it's not breaking the rules, try to get advice.

8) Be prepared to analyze, to break things down into their parts and to declare the relations between those parts. Remember, however, that there is no unique division of something into its "parts."

9) Be prepared to scrap conventional definitions, assumptions, notations, and assignment of "parts."

10) If the problem you're solving is written out, be sure to interpret the meaning of the words in context.

11) Don't be afraid to ask yourself questions.

12) Determine the minimum number of things left for you to know to finish solving the problem.

13) Try to solve a simpler, related problem.

14) Try to solve a more general problem by dropping one or more of the constraints of the problem.

15) If you can, divide a large problem into tractable subproblems. Reduce to cases.

16) Put the problem aside for a day or two.

17) Brainstorm for ideas, even wild ones. Determine which approaches will definitely *not* work.

18) Don't overlook the obvious.

19) Ask yourself what the next best thing is.

20) Guess.

21) If you know the answer but not the proof, work backward from the answer.

22) Look for metaphors from other fields, such as when Rutherford likened the atom to the solar system.

23) Study a variety of 'related' fields.

24) Try to map the problem into a simpler domain of analysis.

25) If your intuition tells you that there's got to be a better way to do something, there probably is.

26) Avoid limiting the applicability of an object as suggested by its commonly accepted name. Try to MacGyver the situation.

27) If you know all (or at least some) of the possible solutions, you can list each one (perhaps in a table) then test them one by one.

28) Draw a graphic or a flowchart of the situation. Reveal relationships, inter-relationships, contingencies, and/or barriers to progress.

29) One of the first requirements of finding a solution is to know precisely what constitutes a solution to the problem. If there's some fuzziness about it, search for a clarification. Does the solution have to be optimal or just 'good enough'?

30) Exploit symmetry.[1]

The above list is not exhaustive. Perhaps you can appreciate the discursive nature of the field as represented by the above heuristics, which range from the psychology of problem solving to the twilight of human creativity.

3 General Heuristics

I now want to give some steps to becoming a better problem solver.

a) Think of yourself as a problem solver. After succeeding in solving a specific problem, intuit the general heuristics you used subconsciously. To do that you may have to carefully analyze what you have done.

b) Many good problem-solving techniques are used implicitly or explicitly in TV programs and novels. Look for them. There is a similarity between being a good problem solver and being a good detective.

c) Study logic and epistemology (the formal theory of knowledge). Avoid logical fallacies.

d) Solve logic games and puzzles.

e) Study thoroughly the problem you need to solve. Be prepared to do research on others' attempts and approaches.

f) Always look for a better way.

g) Increase your vocabulary.

h) Read what other have written on heuristics.

i) Have a large number of problems-to-be-solved cooking on the back burners of your mind.

[1] I got this one from https://triz-journal.com/heuristics-solving-technical-problems-theory-derivation-application.

j) Don't neglect to master the fundamentals. The shovel that digs dirt can also dig gold.

k) Always ask yourself if the principle you've just learned is a special case of a more general principle. Ask yourself if the general principle you've just learned solves or implies an important special case.

l) Always reverse metaphors. For example, if atoms are like stars, in what sense can stars be likened to atoms. (Warning: If you tend to judge this problem as trivial and uninteresting, you haven't thought long enough on it. Sometimes the difference between those who make interesting discoveries and those that don't is merely a matter of attitude and perseverance.)

m) Categorize. Strive for *concinnity*.

Lastly, some negative heuristics to avoid: Avoid counter-productive rules of thumb or dogmas. For example, it may sound reasonable to "Question everything." But think about it carefully. Could you really get anything done if you actually questioned everything?

Avoid being quixotic – idealistic to a fault, impractical. Ideals can be good, but sometimes compromises are needed.