The Intermediate Value Theorem Proof – in a Flowchart

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Abstract

The goal of the short paper is to prove the Intermediate Value Theorem with the aid of a flowchart.

1 The Theorem

Let f be a continuous function on the closed interval $[a, b]$, where $f(a) \neq f(b)$. Then, f will take on all values between $f(a)$ and $f(b)$. What this means is that for any k chosen such that $f(a) < k < f(b)$, then there exists a c in (a, b) such that $f(c) = k$.

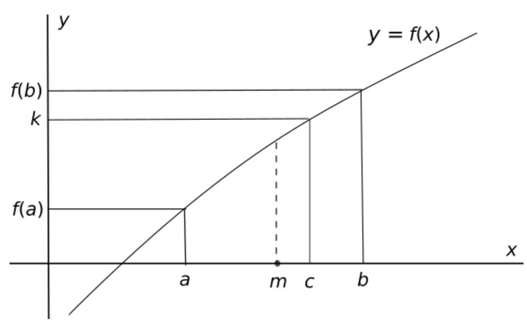

Figure 1. The graph depicts a monotonically increasing function $f(x)$ between points a and b. The theorem claims that if one picks a point k between $f(a)$ and $f(b)$ on the y-axis, that there must exist a point $c \in (a, b)$ on the x-axis such that $f(c) = k$. The point m is the midpoint between a and b.

We will need a result from real analysis:

Lemma: The Monotone Convergence theorem

Every bounded, monotonically increasing (decreasing) sequence of real numbers converges to its limit.

Proof to IVT:

Let k be some arbitrary value between $f(a)$ and $f(b)$. Without loss of generality, we will concern ourselves with an interval about k on the y-axis that is strictly increasing. This is guaranteed because f is assumed to be continuous and increasing.[1]

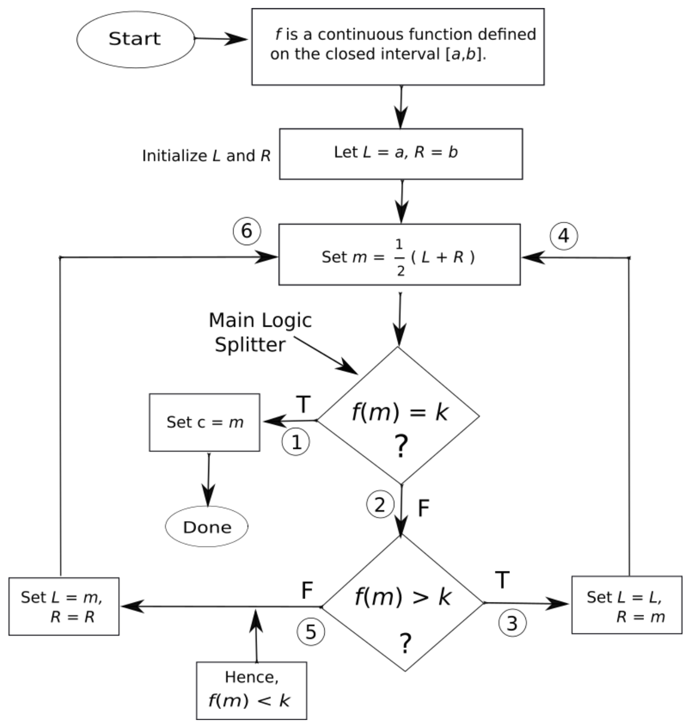

Figure 2. The flowchart reveals one of two fates must occur: Either the main loop is terminated at 'Exit 1' in a finite number of trials and c is found directly, or the loop never terminates, leaving us with a decreasing, bounded sequence on the right, and an increasing, bounded sequence on the left – the limit of both being the point $c \in (a, b)$ that we seek.

Each time the bottom diamond is exited at route 3, it adds another element to the decreasing sequence; and each time the bottom diamond is exited at route 5, it adds another element to the increasing sequence. So, either the main logic splitter is exited at 1 in a finite number of steps, and the answer is the current value of c, or the main loop is never exited and both sequences approach each other at the same limit point c.

[1] If it were instead decreasing, the proof would be similar.