Notes from Page 40

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1 Introduction

On Page 40 of NFCM [1], we find the equation

$$\mathbf{a}\cdot\mathbf{A}_r = \tfrac{1}{2}(\mathbf{a}\mathbf{A}_r - (-1)^r\mathbf{A}_r\mathbf{a})\,, \tag{1}$$

and the result

$$\mathbf{a}\cdot\mathbf{A}_r = (-1)^{r+1}\mathbf{A}_r\cdot\mathbf{a}\,. \tag{2}$$

To show this result, add the relation

$$\mathbf{A}_r\cdot\mathbf{a} = \tfrac{1}{2}(\mathbf{A}_r\mathbf{a} - (-1)^r\mathbf{a}\mathbf{A}_r)\,. \tag{3}$$

2 Demonstration

Now,

$$\begin{aligned}(-1)^{r+1}\mathbf{A}_r\cdot\mathbf{a} &= \tfrac{1}{2}[(-1)^{r+1}\mathbf{A}_r\mathbf{a} - (-1)^{2r+1}\mathbf{a}\mathbf{A}_r) &\text{(4a)}\\ &= \tfrac{1}{2}[\,-(-1)^r\mathbf{A}_r\mathbf{a} + \mathbf{a}\mathbf{A}_r) &\text{(4b)}\\ &= \tfrac{1}{2}[\mathbf{a}\mathbf{A}_r - (-1)^r\mathbf{A}_r\mathbf{a}) &\text{(4c)}\\ &= \mathbf{a}\cdot\mathbf{A}_r\,. &\text{(4d)}\end{aligned}$$

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.