Notes from Page 40b

P. Reany

December 17, 2024

1 Stuff to demonstrate

On Page 40 of NFCM [1], we find the equation

$$\mathbf{abc} = -\mathbf{bac} + 2\mathbf{a}\cdot\mathbf{bc} \tag{1}$$
$$= -\mathbf{b}(-\mathbf{ca} + 2\mathbf{a}\cdot\mathbf{c}) + 2\mathbf{a}\cdot\mathbf{bc}\,. \tag{2}$$

$$\mathbf{abc} - \mathbf{bca} = 2(\mathbf{a}\cdot\mathbf{bc} - \mathbf{a}\cdot\mathbf{cb}) \tag{3}$$

So,

$$\mathbf{a}\cdot\mathbf{bc} - \mathbf{a}\cdot\mathbf{cb} = \tfrac{1}{2}(\mathbf{abc} - \mathbf{bca}) \tag{4}$$
$$= \tfrac{1}{2}[\mathbf{ab}\wedge\mathbf{c} + \mathbf{ab}\cdot\mathbf{c} - (\mathbf{b}\cdot\mathbf{ca} + \mathbf{b}\wedge\mathbf{ca})] \tag{5}$$
$$= \tfrac{1}{2}[\mathbf{ab}\wedge\mathbf{c} - \mathbf{b}\wedge\mathbf{ca}] \tag{6}$$
$$= \mathbf{a}\cdot\mathbf{b}\wedge\mathbf{c}\,, \tag{7}$$

by

$$\mathbf{a}\cdot\mathbf{A}_r = \tfrac{1}{2}(\mathbf{aA}_r - (-1)^r\mathbf{A}_r\mathbf{a})\,. \tag{8}$$

Hence,

$$\mathbf{a}\cdot\mathbf{b}\wedge\mathbf{c} = \mathbf{a}\cdot\mathbf{bc} - \mathbf{a}\cdot\mathbf{cb}\,. \tag{9}$$

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.