Notes from Page 43b

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1 Stuff to demonstrate

On Page 43 of NFCM [1],

$$\mathbf{a} \cdot (\mathbf{b} \wedge \mathbf{C}_r) = \mathbf{a} \cdot \mathbf{b} \wedge \mathbf{C}_r - \mathbf{b} \wedge (\mathbf{a} \cdot \mathbf{C}_r)\,, \tag{1}$$

Let us adopt a 'special notation' for this problem only:

$$\mathbf{A}_r = \mathbf{a}_2 \wedge \mathbf{a}_3 \wedge \cdots \wedge \mathbf{a}_r \tag{2a}$$
$$\mathbf{A}_{r-1} = \mathbf{a}_3 \wedge \mathbf{a}_4 \wedge \cdots \wedge \mathbf{a}_r \tag{2b}$$
$$\mathbf{A}_{r-2} = \mathbf{a}_4 \wedge \mathbf{a}_5 \wedge \cdots \wedge \mathbf{a}_r \tag{2c}$$
$$\text{etc}\,. \tag{2d}$$

Now,

$$\begin{aligned}
\mathbf{a} \cdot (\mathbf{a}_1 \wedge \mathbf{A}_r) &= \mathbf{a} \cdot \mathbf{a}_1 \mathbf{A}_r - \mathbf{a}_1 \wedge (\mathbf{a} \cdot \mathbf{A}_r) & (3a)\\
&= \mathbf{a} \cdot \mathbf{a}_1 \mathbf{A}_r - \mathbf{a}_1 \wedge [\mathbf{a} \cdot (\mathbf{a}_2 \wedge \mathbf{A}_{r-1})] & (3b)\\
&= \mathbf{a} \cdot \mathbf{a}_1 \mathbf{A}_r - \mathbf{a}_1 \wedge [\mathbf{a} \cdot \mathbf{a}_2 \mathbf{A}_{r-1} - \mathbf{a}_2 \wedge (\mathbf{a} \cdot \mathbf{A}_{r-1})] & (3c)\\
&= (\mathbf{a} \cdot \mathbf{a}_1)\mathbf{a}_2 \wedge \cdots \wedge \mathbf{a}_r - (\mathbf{a} \cdot \mathbf{a}_2)\mathbf{a}_1 \wedge \mathbf{a}_3 \wedge \cdots \wedge \mathbf{a}_r \\
&\quad + \mathbf{a}_1 \wedge \mathbf{a}_2 \wedge [\mathbf{a} \cdot \mathbf{a}_3 \mathbf{A}_{r-2} - \mathbf{a}_3 \wedge (\mathbf{a} \cdot \mathbf{A}_{r-2})] & (3d)\\
&= (\mathbf{a} \cdot \mathbf{a}_1)\mathbf{a}_2 \wedge \cdots \wedge \mathbf{a}_r - (\mathbf{a} \cdot \mathbf{a}_2)\mathbf{a}_1 \wedge \mathbf{a}_3 \wedge \cdots \wedge \mathbf{a}_r \\
&\quad + (\mathbf{a} \cdot \mathbf{a}_3)\mathbf{a}_1 \wedge \mathbf{a}_2 \wedge \mathbf{a}_4 \wedge \cdots \wedge \mathbf{a}_r \\
&\qquad - \mathbf{a}_1 \wedge \mathbf{a}_2 \wedge \mathbf{a}_3 \wedge (\mathbf{a} \cdot \mathbf{a}_4 \wedge \cdots \wedge \mathbf{A}_r) & (3e)
\end{aligned}$$

etc.

To do: show that, with $0 < r < s$:

$$(\mathbf{A}_r \wedge \mathbf{b}) \cdot \mathbf{C}_s = \mathbf{A}_r \cdot (\mathbf{b} \cdot \mathbf{C}_s)\,. \tag{4}$$

Hint: Start with

$$(\mathbf{A}_r \mathbf{b})\mathbf{C}_s = \mathbf{A}_r(\mathbf{b}\mathbf{C}_s)\,, \tag{5}$$

and take the $s - (r + 1) = (s - 1) - r$ parts, respectively, on the sides of the equation.

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.