Notes from Page 46

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1 Stuff to demonstrate

On or about page 46 of NFCM [1],

$$|\,A\,| \equiv \langle\, A^\dagger A \,\rangle\,. \tag{1}$$

If

$$\mathbf{B} = \mathbf{a} \wedge \mathbf{b}\,, \tag{2}$$

show that

$$|\,\mathbf{B}\,| = [\,\mathbf{a}^2\mathbf{b}^2 - (\mathbf{a}\cdot\mathbf{b})^2]^{1/2}\,. \tag{3}$$

First, we recall that

$$\mathbf{a} \wedge \mathbf{b} = \mathbf{ab} - \mathbf{a}\cdot\mathbf{b}\,. \tag{4}$$

Let us adopt a 'special notation' for this problem only:

$$\begin{aligned}
|\,\mathbf{B}\,|^2 &= \langle\, (\mathbf{ab} - \mathbf{a}\cdot\mathbf{b})^\dagger(\mathbf{ab} - \mathbf{a}\cdot\mathbf{b})\,\rangle && \text{(5a)}\\
&= \langle\, (\mathbf{ba} - \mathbf{a}\cdot\mathbf{b})(\mathbf{ab} - \mathbf{a}\cdot\mathbf{b})\,\rangle && \text{(5b)}\\
&= \langle\, \mathbf{baab} - \mathbf{a}\cdot\mathbf{b}(\mathbf{ab} + \mathbf{ba}) + (\mathbf{a}\cdot\mathbf{b})^2\,\rangle && \text{(5c)}\\
&= \langle\, \mathbf{a}^2\mathbf{b}^2 - (\mathbf{a}\cdot\mathbf{b})^2\,\rangle && \text{(5d)}\\
&= \mathbf{a}^2\mathbf{b}^2 - (\mathbf{a}\cdot\mathbf{b})^2\,. && \text{(5e)}
\end{aligned}$$

To finish, just take the square root.

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.