Showing the Steps on Page 88

P. Reany

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1 Introduction

On page 88 of NFCM [1], we find the equation

$$\frac{e^{\mathbf{i}\theta} - e^{\mathbf{i}\alpha}}{1 - e^{\mathbf{i}\alpha}} = \lambda e^{\mathbf{i}\phi} \,. \tag{1}$$

Our task is to use this to establish the relation

$$e^{2\mathbf{i}\phi} = e^{\mathbf{i}\theta} \,. \tag{2}$$

We begin by eliminating λ , which is accomplished by dividing through by the reverse of the equation. Now, since all elements in the equation commute with each other, we are basically just doing complex algebra. Let me interchange the sides of Eq. (1) and divide through by complex conjugate, to get

$$\frac{\lambda e^{\mathbf{i}\phi}}{\lambda e^{-\mathbf{i}\phi}} = \frac{\frac{e^{\mathbf{i}\theta} - e^{\mathbf{i}\alpha}}{1 - e^{\mathbf{i}\alpha}}}{\frac{e^{-\mathbf{i}\theta} - e^{-\mathbf{i}\alpha}}{1 - e^{-\mathbf{i}\alpha}}} = \frac{\frac{e^{\mathbf{i}\theta} - e^{\mathbf{i}\alpha}}{1 - e^{\mathbf{i}\alpha}}}{\frac{e^{-\mathbf{i}\theta} - e^{-\mathbf{i}\alpha}}{1 - e^{-\mathbf{i}\alpha}}} \frac{1}{\frac{e^{\mathbf{i}\alpha}}}.$$
(3)

Simplifying a bit, we get

$$e^{2\mathbf{i}\phi} = \frac{\frac{e^{\mathbf{i}\theta} - e^{\mathbf{i}\alpha}}{1 - e^{\mathbf{i}\alpha}}}{\frac{e^{\mathbf{i}(\alpha-\theta)} - 1}{e^{\mathbf{i}\alpha} - 1}} = \frac{e^{\mathbf{i}\theta} - e^{\mathbf{i}\alpha}}{1 - e^{\mathbf{i}(\alpha-\theta)}} = e^{\mathbf{i}\theta}\frac{1 - e^{\mathbf{i}(\alpha-\theta)}}{1 - e^{\mathbf{i}(\alpha-\theta)}} = e^{\mathbf{i}\theta} \,. \tag{4}$$

References

 D. Hestenes, New Foundations for Classical Mechanics, 2nd Ed., Kluwer Academic Publishers, 1999.