Problem 7.2a on Page 103

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1 The Problem

On page 103 of NFCM [1], we find problem (7.2a): For any integer k > 1, show that

$$\frac{dF^k}{dt} = \dot{F}F^{k-1} + F\dot{F}F^{k-2} + \dots + F^{k-1}\dot{F}.$$
(1)

2 Solution

We'll perform a proof by induction. For the case k = 2, the result obviously holds due to Eq. (7.6b) in the text. So, our next step is to assume that (1) is true and then show that the equation holds for the (k + 1)st case. Using Eq. (7.6b) again, we get that

$$\frac{dF^{k+1}}{dt} = \frac{d}{dt}(FF^{k})
= \dot{F}F^{k} + F\dot{F}^{k}
= \dot{F}F^{k} + F(\dot{F}F^{k-1} + F\dot{F}F^{k-2} + \dots + F^{k-1}\dot{F})
= \dot{F}F^{k} + F\dot{F}F^{k-1} + F^{2}\dot{F}F^{k-2} + \dots + F^{k}\dot{F}.$$
(2)

So, whereas the terms on the right hand side of (1) have k - 1 factors each of F, the terms on the right-hand side of (2) have k factors each of F and in proper order with respect to the \dot{F} , which is what we needed to prove. Thus we are finished.

References

 D. Hestenes, New Foundations for Classical Mechanics, 2nd Ed., Kluwer Academic Publishers, 1999.