Problem 8.1 on Page 117

P. Reany

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1 The Problem

On page 117 of NFCM [1], we find problem (8.1): Evaluate the derivatives

$$\mathbf{a}\cdot\nabla(\mathbf{x}\times\mathbf{b}) \tag{1a}$$

$$\mathbf{a}\cdot\nabla(\mathbf{x}\cdot\langle A\rangle_r) \tag{1b}$$

$$\mathbf{a}\cdot[\mathbf{x}\cdot(\mathbf{x}\wedge\mathbf{b})] \tag{1c}$$

2 Solutions

Starting with (1a), we have that

$$\begin{aligned}\mathbf{a}\cdot\nabla(\mathbf{x}\times\mathbf{b}) &= \mathbf{a}\cdot\nabla(-i\mathbf{x}\wedge\mathbf{b})\\ &= -\frac{i}{2}\,\mathbf{a}\cdot\nabla(\mathbf{x}\mathbf{b}-\mathbf{b}\mathbf{x})\\ &= -\frac{i}{2}\,(\mathbf{a}\mathbf{b}-\mathbf{b}\mathbf{a})\\ &= -i\,(\mathbf{a}\wedge\mathbf{b})\\ &= \mathbf{a}\times\mathbf{b}\,. \end{aligned}\tag{2}$$

Next we have (1b),

$$\begin{aligned}\mathbf{a}\cdot\nabla(\mathbf{x}\cdot\langle A\rangle_r) &= \mathbf{a}\cdot\nabla\Big[\tfrac{1}{2}(\mathbf{x}\langle A\rangle_r-(-1)^r\langle A\rangle_r\mathbf{x})\Big]\\ &= \tfrac{1}{2}(\mathbf{a}\langle A\rangle_r-(-1)^r\langle A\rangle_r\mathbf{a})\\ &= \mathbf{a}\cdot\langle A\rangle_r\,. \end{aligned}\tag{3}$$

Lastly, we have (1c),

$$\begin{aligned}\mathbf{a}\cdot[\mathbf{x}\cdot(\mathbf{x}\wedge\mathbf{b})] &= \tfrac{1}{2}\mathbf{a}\cdot\nabla\Big[(\mathbf{x}(\mathbf{x}\wedge\mathbf{b})-(-1)^2(\mathbf{x}\wedge\mathbf{b})\mathbf{x}\Big]\\ &= \tfrac{1}{2}\Big[(\mathbf{a}(\mathbf{x}\wedge\mathbf{b})+\mathbf{x}(\mathbf{a}\wedge\mathbf{b})-(-1)^2\big\{(\mathbf{a}\wedge\mathbf{b})\mathbf{x}+(\mathbf{x}\wedge\mathbf{b})\mathbf{a}\big\}\Big]\\ &= \tfrac{1}{2}\big\{(\mathbf{a}(\mathbf{x}\wedge\mathbf{b})-(-1)^2(\mathbf{a}\wedge\mathbf{b})\mathbf{x}\big\}+\tfrac{1}{2}\big\{\mathbf{x}(\mathbf{a}\wedge\mathbf{b})-(-1)^2(\mathbf{a}\wedge\mathbf{b})\mathbf{x}\big\}\\ &= \mathbf{a}\cdot(\mathbf{b}\wedge\mathbf{x})+\mathbf{x}\cdot(\mathbf{a}\wedge\mathbf{b})\,. \end{aligned}\tag{4}$$

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.