Problem 8.2a on Page 117

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1 The Problem

On page 117 of NFCM [1], we find problem (8.2a): Show that

$$\mathbf{a} \cdot \nabla \, r = \mathbf{a} \cdot \hat{\mathbf{r}} \,, \tag{1}$$

where

$$r = | \, \mathbf{x} - \mathbf{x}' \, | \,. \tag{2}$$

2 Solution

We start with

$$\mathbf{a} \cdot \nabla r = \mathbf{a} \cdot \nabla | \, \mathbf{x} - \mathbf{x}' \, | \,. \tag{3}$$

But

$$| \, \mathbf{x} - \mathbf{x}' \, |^2 = (\mathbf{x} - \mathbf{x}')^2 \,. \tag{4}$$

Therefore,

$$\mathbf{a} \cdot \nabla \, | \, \mathbf{x} - \mathbf{x}' \, |^2 = \mathbf{a} \cdot \nabla (\mathbf{x} - \mathbf{x}')^2 \,. \tag{5}$$

Now,

$$\mathbf{a} \cdot \nabla \, | \, \mathbf{x} - \mathbf{x}' \, |^2 = 2 | \, \mathbf{x} - \mathbf{x}' \, | \, \mathbf{a} \cdot \nabla \, | \, \mathbf{x} - \mathbf{x}' \, | = 2r \, \mathbf{a} \cdot \nabla \, r \,. \tag{6}$$

Whereas

$$\begin{aligned} \mathbf{a} \cdot \nabla \, (\mathbf{x} - \mathbf{x}')^2 &= [\, \mathbf{a} \cdot \nabla (\mathbf{x} - \mathbf{x}') \,](\mathbf{x} - \mathbf{x}') + (\mathbf{x} - \mathbf{x}') \, \mathbf{a} \cdot \nabla \, (\mathbf{x} - \mathbf{x}') \\ &= \mathbf{a}(\mathbf{x} - \mathbf{x}') + (\mathbf{x} - \mathbf{x}')\mathbf{a} \\ &= 2\mathbf{a} \cdot \mathbf{r} \,. \end{aligned} \tag{7}$$

On setting these last two results equal to each other, we get

$$2r \, \mathbf{a} \cdot \nabla \, r = 2\mathbf{a} \cdot \mathbf{r} \,, \tag{8}$$

which, on further simplification, gives us (1).

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.