# Problem 8.2d on Page 117

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## 1 The Problem

On page 117 of NFCM [1], we find problem (8.2d): Show that

$$\mathbf{a}\cdot\nabla(\hat{\mathbf{r}}\wedge\mathbf{a}) = \frac{\hat{\mathbf{r}}\cdot\mathbf{a}\,\mathbf{a}\wedge\hat{\mathbf{r}}}{r}\,, \tag{1}$$

where

$$\mathbf{r} = \mathbf{x}-\mathbf{x}' \quad \text{and} \quad r = |\,\mathbf{x}-\mathbf{x}'\,|\,. \tag{2}$$

## 2 Lemmas (previously proved results)

$$\mathbf{a}\cdot\nabla\,\mathbf{r} = \mathbf{a}\,, \tag{3a}$$

$$\mathbf{a}\cdot\nabla\,\hat{\mathbf{r}} = \frac{\hat{\mathbf{r}}\,\hat{\mathbf{r}}\wedge\mathbf{a}}{r}\,. \tag{3b}$$

## 3 Solution

We start with

$$\begin{aligned}
\mathbf{a}\cdot\nabla\,(\hat{\mathbf{r}}\wedge\mathbf{a}) &= \tfrac{1}{2}\,\mathbf{a}\cdot\nabla\,(\hat{\mathbf{r}}\mathbf{a}-\mathbf{a}\hat{\mathbf{r}}) \\
&= \tfrac{1}{2}\Big[\frac{\hat{\mathbf{r}}\,\hat{\mathbf{r}}\wedge\mathbf{a}\,\mathbf{a}}{r} - \frac{\mathbf{a}\,\hat{\mathbf{r}}\,\hat{\mathbf{r}}\wedge\mathbf{a}}{r}\Big] \\
&= -\tfrac{1}{2}\Big[\frac{\hat{\mathbf{r}}\mathbf{a}\,\hat{\mathbf{r}}\wedge\mathbf{a}}{r} + \frac{\mathbf{a}\,\hat{\mathbf{r}}\,\hat{\mathbf{r}}\wedge\mathbf{a}}{r}\Big] \\
&= -\tfrac{1}{2}\frac{(\hat{\mathbf{r}}\mathbf{a}+\mathbf{a}\,\hat{\mathbf{r}})\,\hat{\mathbf{r}}\wedge\mathbf{a}}{r} \\
&= \frac{\hat{\mathbf{r}}\cdot\mathbf{a}\,\mathbf{a}\wedge\hat{\mathbf{r}}}{r}\,.
\end{aligned} \tag{4}$$

## References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.