

# Problem 8.2e on Page 117

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## 1 The Problem

On page 117 of NFCM [1], we find problem (8.2e): Show that

$$\mathbf{a} \cdot \nabla |\hat{\mathbf{r}} \wedge \mathbf{a}| = -\frac{\hat{\mathbf{r}} \cdot \mathbf{a} |\hat{\mathbf{r}} \wedge \mathbf{a}|}{r}, \quad (1)$$

where

$$\mathbf{r} = \mathbf{x} - \mathbf{x}' \quad \text{and} \quad r = |\mathbf{x} - \mathbf{x}'|. \quad (2)$$

## 2 Lemmas (previously proved results)

$$\mathbf{a} \cdot \nabla (\hat{\mathbf{r}} \wedge \mathbf{a}) = \frac{\hat{\mathbf{r}} \cdot \mathbf{a} \mathbf{a} \wedge \hat{\mathbf{r}}}{r}. \quad (3a)$$

## 3 Solution

As  $|\hat{\mathbf{r}} \wedge \mathbf{a}|^2 = -(\hat{\mathbf{r}} \wedge \mathbf{a})^2$ , we start with

$$\mathbf{a} \cdot \nabla |\hat{\mathbf{r}} \wedge \mathbf{a}|^2 = 2|\hat{\mathbf{r}} \wedge \mathbf{a}| \mathbf{a} \cdot \nabla |\hat{\mathbf{r}} \wedge \mathbf{a}|, \quad (4)$$

and then

$$\begin{aligned} -\mathbf{a} \cdot \nabla (\hat{\mathbf{r}} \wedge \mathbf{a})^2 &= -[\mathbf{a} \cdot \nabla (\hat{\mathbf{r}} \wedge \mathbf{a})] (\hat{\mathbf{r}} \wedge \mathbf{a}) - (\hat{\mathbf{r}} \wedge \mathbf{a}) \mathbf{a} \cdot \nabla (\hat{\mathbf{r}} \wedge \mathbf{a}) \\ &= -\left[ \frac{\hat{\mathbf{r}} \cdot \mathbf{a} \mathbf{a} \wedge \hat{\mathbf{r}}}{r} \right] (\hat{\mathbf{r}} \wedge \mathbf{a}) - (\hat{\mathbf{r}} \wedge \mathbf{a}) \frac{\hat{\mathbf{r}} \cdot \mathbf{a} \mathbf{a} \wedge \hat{\mathbf{r}}}{r} \\ &= -2 \frac{\hat{\mathbf{r}} \cdot \mathbf{a} |\hat{\mathbf{r}} \wedge \mathbf{a}|^2}{r}. \end{aligned} \quad (5)$$

On equating these last two RHS's,

$$2|\hat{\mathbf{r}} \wedge \mathbf{a}| \mathbf{a} \cdot \nabla |\hat{\mathbf{r}} \wedge \mathbf{a}| = -2 \frac{\hat{\mathbf{r}} \cdot \mathbf{a} |\hat{\mathbf{r}} \wedge \mathbf{a}|^2}{r}. \quad (6)$$

On solving this for  $\mathbf{a} \cdot \nabla |\hat{\mathbf{r}} \wedge \mathbf{a}|$ , we get (1).

## References

- [1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.