Problems 8.2k and 8.2l on Page 118

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1 The Problems

On page 118 of NFCM [1], we find problems (8.2k) and (8.2l), but first we do (8.2k): Show that

$$\mathbf{a}\cdot\nabla\,\mathbf{r}^{2k}=2k\,\mathbf{a}\cdot\mathbf{r}\,\mathbf{r}^{2(k-1)}\quad\text{where}\quad\mathbf{r}=\mathbf{x}-\mathbf{x}'\quad\text{and}\quad r=|\,\mathbf{x}-\mathbf{x}'\,|\,. \tag{1}$$

2 Lemmas

$$\mathbf{a}\cdot\nabla\,\mathbf{r}^{2}=2\,\mathbf{a}\cdot\mathbf{r}\,, \tag{2a}$$

$$\mathbf{a}\cdot\nabla\,\mathbf{r}=\mathbf{a}\,, \tag{2b}$$

$$\mathbf{a}\cdot\mathbf{r}\,\mathbf{r}^{2(k-1)}=\mathbf{a}\cdot\mathbf{r}\,\mathbf{r}^{2k-2}\hat{\mathbf{r}}\,\hat{\mathbf{r}}=\mathbf{a}\cdot\hat{\mathbf{r}}\,\mathbf{r}^{2k-2}\mathbf{r}\,\hat{\mathbf{r}}=\mathbf{a}\cdot\hat{\mathbf{r}}\,\mathbf{r}^{2k-1}\,\hat{\mathbf{r}}\,. \tag{2c}$$

3 Solution to (8.2k)

Now,

$$\mathbf{a}\cdot\nabla\,\mathbf{r}^{2k}=\mathbf{a}\cdot\nabla\,(\mathbf{r}^{2})^{k}\,. \tag{3}$$

Thus, we shall take the differential of k factors of $\mathbf{r}^2$, each factor being a scalar, hence

$$\mathbf{a}\cdot\nabla\,\mathbf{r}^{2k}=2k\,\mathbf{a}\cdot\mathbf{r}\,(\mathbf{r}^{2})^{(k-1)}=2k\,\mathbf{a}\cdot\mathbf{r}\,\mathbf{r}^{2(k-1)}\,. \tag{4}$$

4 Problem and Solution to (8.2l)

Show that

$$\mathbf{a}\cdot\nabla\,\mathbf{r}^{2k+1}=\mathbf{r}^{2k}\,(\mathbf{a}+2k\mathbf{a}\cdot\hat{\mathbf{r}}\,\hat{\mathbf{r}})\,, \tag{5}$$

where we will use the result of the last problem. Thus,

$$\begin{aligned}\mathbf{a}\cdot\nabla\,\mathbf{r}^{2k+1}&=\mathbf{a}\cdot\nabla\,\mathbf{r}^{2k}\,\mathbf{r}\\&=[\,\mathbf{a}\cdot\nabla\,\mathbf{r}^{2k}\,]\,\mathbf{r}+\mathbf{r}^{2k}(\mathbf{a}\cdot\nabla\mathbf{r})\\&=[\,2k\,\mathbf{a}\cdot\mathbf{r}\,\mathbf{r}^{2(k-1)}\,\hat{\mathbf{r}}\,\hat{\mathbf{r}}\,]\,\mathbf{r}+\mathbf{r}^{2k}\,\mathbf{a}\\&=\mathbf{r}^{2k}[\,2k\,\mathbf{a}\cdot\mathbf{r}\,\hat{\mathbf{r}}\,]+\mathbf{r}^{2k}\,\mathbf{a}\\&=\mathbf{r}^{2k}\,(\mathbf{a}+2k\,\mathbf{a}\cdot\hat{\mathbf{r}}\,\hat{\mathbf{r}})\end{aligned} \tag{6}$$

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.