Problem 2.4 on Page 134

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1 The Problem

On page 134 of NFCM [1], we find Problem (2.4): Use Equations (2.18) and (2.19) to determine the maximum horizontal range for a projectile with initial speed v_0 fired on targets on plateau with vertical elevation y above the firing pad.

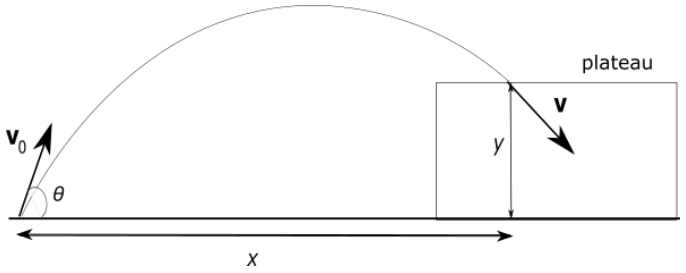

Figure 1. We're looking for a solution of maximum horizontal range. It turns out that $\mathbf{v}_0$ and $\mathbf{v}$ must be orthogonal.

2 Solution

For reference, the afformentioned texbook equations are:

$$2\,\mathbf{g}\cdot\mathbf{r} = v^2 - v_0^2\,, \tag{2.18}$$

and

$$\mathbf{g}\wedge\mathbf{r} = \mathbf{v}\wedge\mathbf{v}_0\,. \tag{2.19}$$

So, from (2.18), we have that

$$y = -\hat{\mathbf{g}}\cdot\mathbf{r} = \frac{v^2 - v_0^2}{-2g}\,. \tag{1}$$

On solving for v, we get

$$v = [\,v_0^2 - 2gy\,]^{1/2}\,. \tag{2}$$

Now, with $\mathbf{x} = x\,\hat{\mathbf{x}}$

$$\begin{aligned}\mathbf{g}\wedge\mathbf{r} = \mathbf{g}\mathbf{x} = gx\mathbf{i} &= \mathbf{v}\wedge\mathbf{v}_0\\ &= \hat{\mathbf{v}}\wedge\hat{\mathbf{v}}_0\,v\,v_0\\ &= \hat{\mathbf{v}}\wedge\hat{\mathbf{v}}_0\,[\,v_0^2 - 2gy\,]^{1/2}\,v_0\end{aligned} \tag{3}$$

Hence,

$$x\mathbf{i} = \hat{\mathbf{v}} \wedge \hat{\mathbf{v}}_0 \left[v_0^2 - 2gy \right]^{1/2} v_0 \,. \tag{4}$$

Now, to maximize x, given y and v_0, we must make $\hat{\mathbf{v}} \wedge \hat{\mathbf{v}}_0$ as large as it can be, which means that $\hat{\mathbf{v}} \wedge \hat{\mathbf{v}}_0 = \mathbf{i}$. This means that at maximum range, $x_{\max}$, $\mathbf{v}$ and $\mathbf{v}_0$ are perpendicular to each other.

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.