Problem 2.6 on Page 134

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May 26, 2021

1 The Problem

On page 134 of NFCM [1], we find Problem (2.6): From Eqs. (2.15) and (2.16) obtain

$$\mathbf{v} \wedge \mathbf{g} = \mathbf{v}_0 \wedge \mathbf{g} \,. \tag{1}$$

$$\mathbf{v} \wedge \mathbf{r} = \mathbf{r} \wedge \mathbf{v}_0 \,. \tag{2}$$

Then solve these to get

$$\mathbf{v} = \left(\frac{\mathbf{v}_0 \wedge \mathbf{r}}{\mathbf{r} \wedge \mathbf{g}} \right) \mathbf{g} + \left(\frac{\mathbf{v}_0 \wedge \mathbf{g}}{\mathbf{r} \wedge \mathbf{g}} \right) \mathbf{r} \,. \tag{3}$$

2 Solution

To show this, I'll use the method of undetermined coefficients. We begin with

$$\mathbf{v} = \alpha \, \mathbf{g} + \beta \mathbf{r} \,, \tag{4}$$

where, of course, the coefficients are to be determined. On wedging (4) through by $\mathbf{r}$, we obtain

$$\mathbf{v} \wedge \mathbf{r} = \alpha \, \mathbf{g} \wedge \mathbf{r} \,. \tag{5}$$

On solving for α and using (2), we get

$$\alpha = \left(\frac{\mathbf{v} \wedge \mathbf{r}}{\mathbf{g} \wedge \mathbf{r}} \right) = \left(\frac{\mathbf{r} \wedge \mathbf{v}_0}{\mathbf{g} \wedge \mathbf{r}} \right) = \left(\frac{\mathbf{v}_0 \wedge \mathbf{r}}{\mathbf{r} \wedge \mathbf{g}} \right) \,. \tag{6}$$

On solving for β, by similar means, and using (1), we get

$$\beta = \left(\frac{\mathbf{v}_0 \wedge \mathbf{g}}{\mathbf{r} \wedge \mathbf{g}} \right) \,. \tag{7}$$

Substituting these last two results into (4), we obtain (3).

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.