Problem 2.7 on Page 134

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1 The Problem

On page 134 of NFCM [1], we find Problem (2.7): Determine the area swept out in time t by the displacement vector of a particle with constant acceleration $\mathbf{g}$ and initial velocity $\mathbf{v}_0$.

2 Solution

Our starting equation for the swept out area is

$$\mathbf{A} = \tfrac{1}{2}\int_0^t \mathbf{r} \wedge d\mathbf{r}\,. \tag{1}$$

On substituting $\mathbf{v}\,dt$ for $d\mathbf{r}$, we get

$$\mathbf{A} = \tfrac{1}{2}\int_0^t \mathbf{r} \wedge \mathbf{v}dt\,. \tag{2}$$

From Eq. (2.3) on page 126, we have

$$\mathbf{v} = \mathbf{g}t + \mathbf{v}_0\,, \tag{3}$$

and from Eq. (2.4) on page 126, we have

$$\mathbf{r} = \tfrac{1}{2}\mathbf{g}t^2 + \mathbf{v}_0 t\,. \tag{4}$$

Therefore,

$$\begin{aligned} \mathbf{r} \wedge \mathbf{v} &= (\tfrac{1}{2}\mathbf{g}t^2 + \mathbf{v}_0 t) \wedge (\mathbf{g}t + \mathbf{v}_0) \\ &= \tfrac{1}{2}\mathbf{v}_0 \wedge \mathbf{g}t^2\,. \end{aligned} \tag{5}$$

On substituting this value into (2), we get

$$\mathbf{A} = \tfrac{1}{2}\int_0^t \tfrac{1}{2}\mathbf{v}_0 \wedge \mathbf{g}\,t^2 dt = \frac{1}{12}\mathbf{v}_0 \wedge \mathbf{g}\,t^3\,. \tag{6}$$

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.