Complete Computation on Page 162

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1 The Problem

On page 162 of NFCM [1], we find an integral computation. Let's perform the detailed integration here. Note that some of the computations involving the integration of complex functions have already been done for us. (See the integrals just after Eq. (7.18) in the text.)

So, we are instructed to start with Eq. (7.5), which is as follows:

$$\mathbf{v} = R^\dagger \left[\mathbf{v}_0 + \int_0^\infty R\,\mathbf{g} R^\dagger dt \right] R\,, \tag{1}$$

where $R = e^{\frac{1}{2} i\boldsymbol{\omega} t} = e^{\frac{1}{2}\mathbf{i}\omega t}$. For $\mathbf{g}$ in (1), we use Eq. (7.18)

$$\mathbf{g} = \mathbf{g}_0 \cos\omega' t\,. \tag{2}$$

2 Solution

Now we just do it!

$$\begin{aligned}
\mathbf{v} &= R^\dagger \mathbf{v}_0 R + R^\dagger \left[\int_0^\infty R\,(\mathbf{g}_0 \cos\omega' t) R^\dagger dt \right] R \\
&= R^\dagger \mathbf{v}_0 R + R^\dagger \left[\int_0^\infty R\,(\mathbf{g}_{0\|} + \mathbf{g}_{0\perp}) \cos\omega' t R^\dagger dt \right] R \\
&= R^\dagger \mathbf{v}_0 R + R^\dagger \left[\int_0^\infty \mathbf{g}_{0\|} \cos\omega' t + \mathbf{g}_{0\perp}\, \cos\omega' t\, e^{-i\boldsymbol{\omega} t} dt \right] R \\
&= R^\dagger \mathbf{v}_0 R + R^\dagger \left[\mathbf{g}_{0\|} \frac{\sin\omega' t}{\omega'} + \mathbf{g}_{0\perp}\, e^{-i\boldsymbol{\omega} t} \Big(\frac{\omega' \sin\omega' t - \mathbf{i}\omega \cos\omega' t}{\omega'^2 - \omega^2} \Big) \right] R \\
&= R^\dagger \mathbf{v}_0 R + \mathbf{g}_{0\|} \frac{\sin\omega' t}{\omega'} + \mathbf{g}_{0\perp} \Big(\frac{\omega' \sin\omega' t - \mathbf{i}\omega \cos\omega' t}{\omega'^2 - \omega^2} \Big) \\
&= R^\dagger (\mathbf{v}_{0\|} + \mathbf{v}_{0\perp}) R + \mathbf{g}_{0\|} \frac{\sin\omega' t}{\omega'} + \mathbf{g}_{0\perp} \Big(\frac{\omega' \sin\omega' t - \mathbf{i}\omega \cos\omega' t}{\omega'^2 - \omega^2} \Big) \\
&= \mathbf{g}_{0\perp} \Big(\frac{\omega' \sin\omega' t - i\boldsymbol{\omega} \cos\omega' t}{\omega'^2 - \omega^2} \Big) + \mathbf{g}_{0\|} \frac{\sin\omega' t}{\omega'} + \mathbf{v}_{0\perp} e^{i\boldsymbol{\omega} t} + + \mathbf{v}_{0\|}\,.
\end{aligned} \tag{3}$$

Note that we have interchanged $i\boldsymbol{\omega}$ and $\mathbf{i}\omega$ as needed.

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.