Trig Result on Page 209

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1 Introduction

On page 209 of NFCM [1], we find the result (Eq. (3.16))

$$\tan\theta = \frac{\lambda \sin\alpha \cos\alpha}{1 - \lambda^2 \sin^2\alpha}\,, \tag{1}$$

which we shall prove (almost).

2 Proof:

From Eq. (3.13) we have:

$$\boldsymbol{\epsilon} = \mathbf{i}\hat{\mathbf{v}}\lambda \sin\alpha - \hat{\mathbf{r}}\,. \tag{2}$$

From Eq. (3.11) we have the angle α defined

$$\hat{\mathbf{r}}\hat{\mathbf{v}} = e^{\mathbf{i}\alpha}\,. \tag{3}$$

Multipying (2) on the right by $\hat{\mathbf{r}}$, we get

$$\begin{aligned} \boldsymbol{\epsilon}\hat{\mathbf{r}} &= \lambda \sin\alpha\, \mathbf{i}\hat{\mathbf{v}}\hat{\mathbf{r}} - 1 \\ &= \lambda \sin\alpha\, \mathbf{i}\,[\cos\alpha - \mathbf{i}\sin\alpha] - 1 \\ &= (\lambda \sin^2\alpha - 1) + \mathbf{i}(\lambda \sin\alpha\, \cos\alpha)\,, \end{aligned} \tag{4}$$

which agrees with Eq. (3.14). We also have the angle θ defined for us by

$$\boldsymbol{\epsilon}\hat{\mathbf{r}} \equiv \epsilon e^{\mathbf{i}\theta} = \epsilon\cos\theta + \mathbf{i}\epsilon\sin\theta\,. \tag{5}$$

This gives us

$$\epsilon\cos\theta = \lambda\sin^2\alpha - 1 \qquad \text{and} \qquad \epsilon\sin\theta = \lambda\sin\alpha\,\cos\alpha\,. \tag{6}$$

Hence, to form the tangent of θ, we get

$$\tan\theta = \frac{\lambda \sin\alpha \cos\alpha}{\lambda \sin^2\alpha - 1}\,, \tag{7}$$

which disagrees with the given result by an overall minus sign.

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.