Problem 3.2 on Page 213

P. Reany

June 26, 2021

1 The Problem

On page 213 of NFCM [1], we find problem (3.2): The turning points of an orbit are defined by the condition $\mathbf{v} \cdot \mathbf{r} = 0$. Show that for a turning point, Equation (3.3) [pg. 205] gives the relation

$$\mathbf{r} = \frac{k}{2E}(\boldsymbol{\epsilon} - \hat{\mathbf{r}})\,. \tag{1}$$

[Partial answer]

Equation (3.2) is

$$\mathbf{L} = m\mathbf{r}\wedge\mathbf{v}\,. \tag{3.2}$$

But we are here interested in those orbit points satifying the relation

$$\mathbf{r}_T \cdot \mathbf{v}_T = 0\,, \tag{2}$$

where the T subscript stands for a turning point. But this condition makes $\mathbf{L}$ in (3.2) even easier:

$$\mathbf{L} = m\mathbf{r}_T\mathbf{v}_T\,. \tag{3.2'}$$

However, from Equation (3.3):

$$\mathbf{L}\mathbf{v}_T = k(\hat{\mathbf{r}}_T+\boldsymbol{\epsilon})\,, \tag{3.3}$$

which becomes

$$m\mathbf{r}_T\mathbf{v}_T^2 = k(\hat{\mathbf{r}}_T + \boldsymbol{\epsilon})\,. \tag{3}$$

But

$$2E = m\mathbf{v}_T^2 - \frac{2k}{r}\,, \tag{4}$$

which we rewrite as

$$m\mathbf{v}_T^2 = 2E + \frac{2k}{r}\,, \tag{5}$$

On substituting this last result into (3), we get

$$\mathbf{r}_T(2E + \frac{2k}{r}) = k(\hat{\mathbf{r}}_T + \boldsymbol{\epsilon})\,, \tag{6}$$

which becomes

$$2E\mathbf{r}_T = k(-\hat{\mathbf{r}}_T + \boldsymbol{\epsilon})\,, \tag{7}$$

and finally,

$$\mathbf{r}_T = \frac{k}{2E}(\boldsymbol{\epsilon} - \hat{\mathbf{r}}_T)\,. \tag{8}$$

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.