

Problem 1.2 on Page 259

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1 Problem

On page 259 of NFCM [1], we find Problem (1.2):

For f a linear function, α a scalar, and $\mathbf{X} = \langle \mathbf{X} \rangle_{\bar{k}}$, show that

$$\underline{f}(\alpha \mathbf{X}) = \alpha \underline{f}(\mathbf{X}) . \quad (1)$$

2 Solution

Let $\mathbf{X} = \bigwedge_{i=1}^k \mathbf{x}_i$. Now let $\mathbf{x}'_i = \mathbf{x}_i$ for $i \in [2..k]$ and let $\mathbf{x}'_1 = \alpha \mathbf{x}_1$. Then

$$\alpha \mathbf{X} = \alpha \bigwedge_{i=1}^k \mathbf{x}_i = \alpha \mathbf{x}_1 \wedge [\bigwedge_{i=2}^k \mathbf{x}_i] = \mathbf{x}'_1 \wedge [\bigwedge_{i=2}^k \mathbf{x}'_i] = \bigwedge_{i=1}^k \mathbf{x}'_i = \mathbf{X}' \quad (2)$$

Therefore,

$$\begin{aligned} \underline{f}(\alpha \mathbf{X}) &= \underline{f}(\mathbf{X}') \\ &= \underline{f}(\bigwedge_{i=1}^k \mathbf{x}'_i) \\ &= \bigwedge_{i=1}^k \underline{f}(\mathbf{x}'_i) \\ &= \bigwedge_{i=1}^k f(\mathbf{x}'_i) \\ &= f(\mathbf{x}'_1) \wedge [\bigwedge_{i=2}^k f(\mathbf{x}'_i)] \\ &= f(\alpha \mathbf{x}_1) \wedge [\bigwedge_{i=2}^k f(\mathbf{x}_i)] \\ &= \alpha f(\mathbf{x}_1) \wedge [\bigwedge_{i=2}^k f(\mathbf{x}_i)] \\ &= \alpha \underline{f}(\bigwedge_{i=1}^k \mathbf{x}_i) \\ &= \alpha \underline{f}(\mathbf{X}) . \end{aligned} \quad (3)$$

References

- [1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.