Problem 1.3 on Page 260

P. Reany

October 8, 2021

1 Problem

On page 260 of NFCM [1], we find Problem (1.3):

Show that

$$(\mathbf{u}\wedge\mathbf{v})\cdot\underline{f}(\mathbf{x}\wedge\mathbf{y})=\bar{f}(\mathbf{u}\wedge\mathbf{v})\cdot(\mathbf{x}\wedge\mathbf{y})\,. \tag{1}$$

2 Solution

Okay,

$$\begin{aligned}(\mathbf{u}\wedge\mathbf{v})\cdot\underline{f}(\mathbf{x}\wedge\mathbf{y}) &= (\mathbf{u}\wedge\mathbf{v})\cdot f(\mathbf{x})\wedge f(\mathbf{y})\\ &= \mathbf{u}\cdot[\mathbf{v}\cdot f(\mathbf{x})\wedge f(\mathbf{y})]\\ &= \mathbf{u}\cdot[\mathbf{v}\cdot f(\mathbf{x})f(\mathbf{y})-\mathbf{v}\cdot f(\mathbf{y})f(\mathbf{x})]\\ &= \mathbf{v}\cdot f(\mathbf{x})\mathbf{u}\cdot f(\mathbf{y})-\mathbf{u}\cdot f(\mathbf{x})\mathbf{v}\cdot f(\mathbf{y})\\ &= \bar{f}(\mathbf{v})\cdot\mathbf{x}\,\bar{f}(\mathbf{u})\cdot\mathbf{y}-\bar{f}(\mathbf{u})\cdot\mathbf{x}\,\bar{f}(\mathbf{v})\cdot\mathbf{y}\\ &= \quad\ldots\qquad\text{(reverse the steps)}\\ &= \bar{f}(\mathbf{u}\wedge\mathbf{v})\cdot(\mathbf{x}\wedge\mathbf{y})\,. \end{aligned}\tag{2}$$

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.