Problem 1.5 on Page 260

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1 Problems

On page 260 of NFCM [1], we find Problem (1.5):
Prove that

(a) $\det(gf) = \det g \det f$,
(b) $\det(f^{-1}) = (\det f)^{-1}$.

2 Solutions

Part (a): First, let $h = gf$, then we begin with (using Eq. (1.15), p. 255))[1]

$$\det h = i^{-1}\, \underline{h}(i)\,. \tag{1}$$

So,

$$\begin{aligned} \det(gf) &= i^{-1}\, \underline{g}\,\underline{f}(i) = i^{-1}\, \underline{g}(\underline{f}(i)) \\ &= i^{-1}\, \underline{g}(ii^{-1}\, \underline{f}(i)) = i^{-1}\, \underline{g}(i \det f) \\ &= (\det f)(i^{-1}\, \underline{g}(i)) = \det f\, \det g\,. \end{aligned} \tag{2}$$

Note: Since g is linear and $\det f$ is a scalar, then $\underline{g}(i \det f) = (\det f)\underline{g}(i)$.

Part (b): This result follows as a corollary to the previous result. We'll begin with the claim that the determinant of the identity operator is unity. Or,

$$\det \mathbb{1} = 1\,. \tag{3}$$

Therefore, since $f^{-1}f = \mathbb{1}$, then, using the result from (a), with $g = f^{-1}$,

$$\det(f^{-1}f) = (\det f^{-1})(\det f) = 1\,. \tag{4}$$

And, therefore

$$\det(f^{-1}) = (\det f)^{-1}\,. \tag{5}$$

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.

[1]Remember, f, g, h are operators only on vectors, whereas $\underline{f}, \underline{g}, \underline{h}$ are operators on vectors, bivectors, or pseudoscalars.