Problem 1.6 on Page 260

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1 Problems

On page 260 of NFCM [1], we find Problem (1.6):

To find the inverse of a linear transformation, Equation (1.22) can always be used, but a more direct approach is often better. Find the inverse of

$$\mathbf{y} = \alpha\mathbf{x} + \mathbf{a}\,\mathbf{b}\cdot\mathbf{x} \tag{1}$$

by solving the algebraic equation $\mathbf{y} = f\mathbf{x}$ for $\mathbf{x}$ as a function of $\mathbf{y}$.

2 Solution

We must replace $\mathbf{b}\cdot\mathbf{x}$ by something in terms of $\mathbf{y}$. To the end, we dot (1) through by $\mathbf{b}$:

$$\mathbf{b}\cdot\mathbf{y} = \alpha\mathbf{b}\cdot\mathbf{x} + \mathbf{b}\cdot\mathbf{a}\,\mathbf{b}\cdot\mathbf{x} \tag{2}$$

Now, we just solve this for $\mathbf{b}\cdot\mathbf{x}$, to get

$$\mathbf{b}\cdot\mathbf{x} = \frac{\mathbf{b}\cdot\mathbf{y}}{\alpha + \mathbf{a}\cdot\mathbf{b}} \tag{3}$$

By substituting this result into (1), we get

$$\mathbf{y} = \alpha\mathbf{x} + \mathbf{a}\frac{\mathbf{b}\cdot\mathbf{y}}{\alpha + \mathbf{a}\cdot\mathbf{b}}\,. \tag{4}$$

Now we can easily solve for $\mathbf{x}$:

$$\mathbf{x} = \alpha^{-1}\Big[\mathbf{y} - \mathbf{a}\frac{\mathbf{b}\cdot\mathbf{y}}{\alpha + \mathbf{a}\cdot\mathbf{b}}\Big]\,. \tag{5}$$

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.