Problems 2.3 on Page 613

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1 Problem 2.3

On page 613 of NFCM [1], we find problem (2.3): In the twin problem, each twin can "watch" the aging of the other continuously by receiving light signals from each other , as indicated in Figure 2.16. Calculate the aging Δt_1 and Δt_2 of the homebody "seen" by the traveler on each leg of his journey. Similarly, calculate and interpret δt_1 and δt_2. Show that the results are consistent withthe round-trip aging given by the time-dilatin formula.

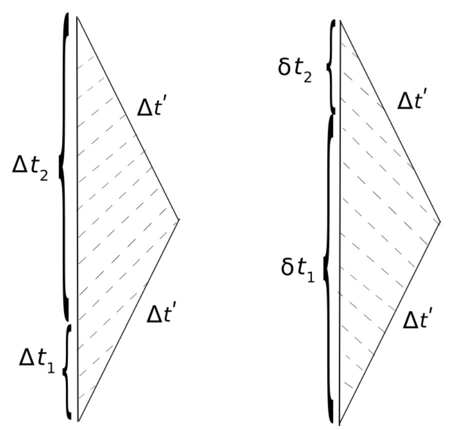

Figure 1. This figure corresponds to Fig. 2.16 (p. 614) in the text.

2 Solution to 2.3

If we look back to Fig. 2.15 in Problem (2.1), we see that

$$t_1 = \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}} t' \,, \tag{1}$$

and

$$t_2 = \left(\frac{1+\beta}{1-\beta}\right)^{\frac{1}{2}} t' \,. \tag{2}$$

Equation (1) compares to the left figure of Fig. 2.16 to give us

$$\Delta t_1 = \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}} \Delta t' \,. \tag{3}$$

Equation (2) compares to the right figure of Fig. 2.16 to give us

$$\delta t_1 = \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}} \Delta t' \,. \tag{4}$$

Once the astronaut twin has completed his acceleration, he can claim to be at rest and the earth moving towards him. Then the roles have been reversed, so that

$$\Delta t' = \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}} \Delta t_2 \,. \tag{5}$$

Equation (2) compares to the right figure of Fig. 2.16 to give us

$$\delta t' = \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}} \delta t_2 \,. \tag{6}$$

To finish, we'll need the result

$$\left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}} + \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}} = 2\gamma \,. \tag{7}$$

From the preceding equaitons we have that

$$\Delta t_1 + \Delta t_2 = \left[\left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}} + \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}}\right] \Delta t' = 2\gamma \Delta t' \,, \tag{8}$$

and

$$\delta t_1 + \delta t_2 = \left[\left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}} + \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}}\right] \Delta t' = 2\gamma \Delta t' \,. \tag{9}$$

References

[1] D. Hestenes, *New Foundations for Classical Mechanics*, 2nd Ed., Kluwer Academic Publishers, 1999.