

# Much Ado About Ellipses, Part 2: Area

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## 1 Introduction

We've only one short task in this article, and that is to find the area of an ellipse. Familiarity with integral calculus is assumed.

## 2 Theorem

The area of an ellipse of semimajor length  $a$  and semiminor length  $b$  is

$$\text{area} = \pi ab. \tag{1}$$

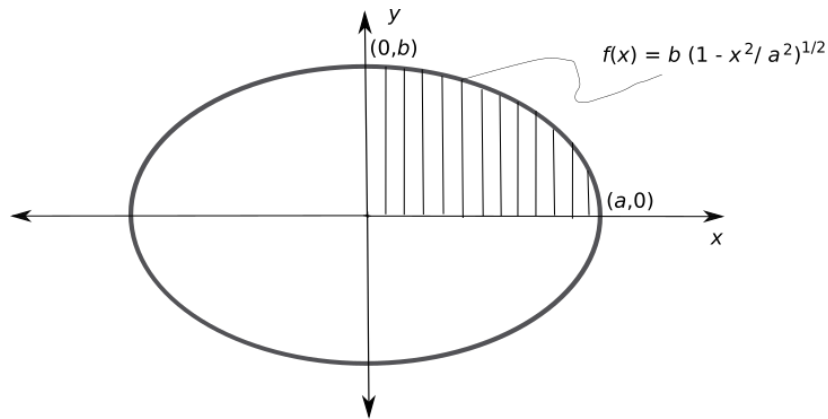


Figure 1. The area of the ellipse is four times the area of its first quadrant contribution.

The standard equation for the ellipse in terms of coordinates is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \tag{2}$$

But this equation is not in the form of a function that we can use to perform a Riemann integration. To fix that, we solve Equation (2) for  $y = f(x)$  only in

the first quadrant, to get

$$f(x) = b\sqrt{1 - x^2/a^2}. \quad (3)$$

Then, performing the integration in the first quadrant, we have that

$$\text{area}_1 = b \int_0^a \sqrt{1 - x^2/a^2} dx, \quad (4)$$

where  $\text{area}_1$  is the area only in the first quadrant.

Making the variable substitution  $z = x/a$ , the integral becomes

$$\text{area}_1 = ab \int_0^1 \sqrt{1 - z^2} dz = \frac{1}{4}\pi ab. \quad (5)$$

The area of the entire ellipse is then

$$\text{area} = \pi ab. \quad (6)$$

I leave it to the interested reader to perform the above integral. The indefinite integral is as follows:

$$\int \sqrt{1 - z^2} dz = \frac{1}{2}z\sqrt{1 - z^2} + \frac{1}{2} \tan^{-1} \frac{z}{\sqrt{1 - z^2}}. \quad (7)$$