Logarithms

P. Reany

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If you're not in love with the Truth, you could be
talked into believing almost anything.
— Author

1 The Theory

No theory as of yet. Perhaps later. By the way, the logarithm identities in this article deal with the logarithm function over the real numbers.

2 Identities

1)

$$\log_a a = 1 \quad \text{and} \quad \log_a 1 = 0\,. \tag{1}$$

2)

$$a^{\log_a x} = x\,. \tag{2}$$

3)

$$\log_a x^b = b \log_a x\,, \tag{3a}$$

$$\log_a \frac{1}{x} = \log_a x^{-1} = -\log_a x \quad (x \neq 0)\,. \tag{3b}$$

4) Converting to a different base c:

$$\log_a x = \frac{\log_c x}{\log_c a}\,. \tag{4}$$

5) A multiplicative inverse:

$$\log_a x \, \log_x a = 1\,. \tag{5}$$

6) The 'natural logarithm':

$$\ln x \equiv \log_e x\,. \tag{6}$$

7) The product and quotient identities (arbitrary base):

$$\log(xy) = \log x + \log y\,, \tag{7a}$$
$$\log(x/y) = \log x - \log y\,, \tag{7b}$$

8) Since

$$\log_a x = \frac{\log_c x}{\log_c a}\,, \tag{8}$$

and for a different base $c' \neq c$:

$$\log_a x = \frac{\log_{c'} x}{\log_{c'} a}\,, \tag{9}$$

then

$$\frac{\log_c x}{\log_c a} = \frac{\log_{c'} x}{\log_{c'} a}\,, \tag{10}$$

In particular, with $c = 10$ and $c' = e$:[1]

$$\frac{\log x}{\log a} = \frac{\ln x}{\ln a}\,. \tag{11}$$

9) An identity with proof.

$$\log_{xy} a = \frac{\log a}{\log xy} = \frac{\log a}{\log x + \log y} \tag{12}$$
$$= \frac{1}{\dfrac{\log x}{\log a} + \dfrac{\log y}{\log a}} = \frac{1}{\log_a x + \log_a y} \tag{13}$$
$$= \frac{1}{\dfrac{1}{\log_x a} + \dfrac{1}{\log_y a}} \tag{14}$$

As simple as rolling off a log.

[1] When a logarithm is shown without a specific base, the base is often, but not always, understood to be base 10. In particular, Wolfram Alpha seems to always write 'log' whether it means base 10 or base e. It will specify in the description which it is. In this article, if no base is specified, consider it to be arbitrary.