

Math Diversion Problem 101

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Programming isn't about what you know;
it's about what you can figure out.

— Chris Pine

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=S64w7YxWoTs>
Title: France | Olympic math question - past exam
SAT MATH QUESTION
Presenter: Kmath addict

1 The Problem

Given the relation

$$5^x + 35^{x/2} = 7^x, \quad (1)$$

find the values of x .

2 The Solution

We begin by converting the given equation to

$$35^{x/2} = 7^x - 5^x, \quad (2)$$

and then squaring

$$35^x = 7^x \cdot 5^x = (7^x - 5^x)^2 \quad (3)$$

$$= (7^x + 5^x)^2 - 4 \cdot 7^x \cdot 5^x. \quad (4)$$

Now, we adjust:

$$5 \cdot 7^x \cdot 5^x = (7^x + 5^x)^2. \quad (5)$$

It may not seem intuitive to take the square root on both sides, but that's the way to a quick finish.

$$\sqrt{5} \cdot 7^{x/2} \cdot 5^{x/2} = 7^x + 5^x. \quad (6)$$

But we can adjust the LHS by going back to (2):

$$\sqrt{5}(7^x - 5^x) = 7^x + 5^x. \quad (7)$$

Now we collect on 7^x and 5^x .

$$(\sqrt{5} - 1)7^x = (\sqrt{5} + 1)5^x, \quad (8)$$

which gives us

$$\frac{7^x}{5^x} = \left(\frac{7}{5}\right)^x = \frac{\sqrt{5} + 1}{\sqrt{5} - 1}. \quad (9)$$

On taking the logarithm of both sides, we get

$$x = \frac{\log \frac{\sqrt{5} + 1}{\sqrt{5} - 1}}{\log \left(\frac{7}{5}\right)} = \frac{\log \frac{(\sqrt{5} + 1)^2}{4}}{\log \left(\frac{7}{5}\right)} = \frac{2 \log \frac{1}{2}(\sqrt{5} + 1)}{\log \left(\frac{7}{5}\right)}. \quad (10)$$