

Math Diversion Problem 102

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Truth, like oil, will in time rise to surface.

— Charlie Chan

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=nL8SN1xaSfc>

Title: Cambridge University Admission Exam Tricks

Presenter: Super Academy

1 The Problem

Given the relations

$$x^2 + xy + y^2 = 96, \quad (1a)$$

$$x + \sqrt{xy} + y = 16, \quad (1b)$$

find the values of $x, y \in \mathbb{R}$.

2 The Solution

We begin by squaring (1b):

$$x^2 + xy + y^2 + 2x\sqrt{xy} + 2xy + 2y\sqrt{xy} = 256. \quad (2)$$

On combining this with (1a), we have that

$$96 + 2x\sqrt{xy} + 2xy + 2y\sqrt{xy} = 256. \quad (3)$$

And this simplifies down to

$$\sqrt{xy} = \frac{160}{32} = 5, \quad (4a)$$

$$xy = 25. \quad (4b)$$

Next, we use (4a) in (1b) to get

$$x + y = 11. \quad (5)$$

Multiplying this through by x and rearranging gives us

$$x^2 - 11x + xy = 0. \tag{6}$$

But we just showed that $xy = 25$, so

$$x^2 - 11x + 25 = 0, \tag{7}$$

which has roots

$$x = \frac{1}{2}(11 \pm \sqrt{21}). \tag{8}$$

Lastly, we calculate y from (5):

$$y = \frac{1}{2}(11 \mp \sqrt{21}). \tag{9}$$