Math Diversion Problem 108

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> You cannot ask us to take sides against arithmetic.
> — Winston Churchill

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=IGUh3Mv7DAI
Title: One Of The Most Difficult Harvard's University Entrance Exam
 Presenter: Maths with Chinwendu
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1 The Problem

Given the relation

$$x^x y^y = 8x^y y^x \,, \tag{1}$$

find the smallest positive integer values of x, y with $x > y$.

2 The Solution

Begin by dividing through by $x^y y^x$:

$$x^{x-y} y^{y-x} = 8 \,. \tag{2}$$

On further simplification,

$$\left(\frac{x}{y}\right)^{x-y} = 8 = 2^3 \,. \tag{3}$$

So, look: Since 8 is such a small number, let's just guess!

$$\frac{x}{y} = 2 \,. \tag{4}$$

But $2^3 = 8$, so

$$x - y = 3 \,. \tag{5}$$

On using this with (4), we get

$$x = 6, \quad y = 3 \,. \tag{6}$$