Math Diversion Problem 113

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> I love it when a plan comes together.
> — Hannibal Smith, *The A-Team*

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=ZQZMu0SraSQ
Title: Can you Pass Harvard University Admission Interview
 Presenter: Enjoy Math
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1 The Problem

Given the relation

$$\sqrt{\frac{4^{20}-2^{21}+1}{2^{20}+2^{11}+1}} = 2^x - 1\,, \tag{1}$$

find the values of x.

2 The Solution

Let's begin by doing a trivial adjustment on the bases so that they are consistent:

$$\sqrt{\frac{2^{40}-2\cdot 2^{20}+1}{2^{20}+2\cdot 2^{10}+1}} = 2^x - 1\,, \tag{2}$$

Now we can do some factoring and then cancelling:

$$\sqrt{\frac{(2^{20}-1)^2}{(2^{10}+1)^2}} = \sqrt{\frac{(2^{10}+1)^2(2^{10}-1)^2}{(2^{10}+1)^2}} \tag{3a}$$

$$= (2^{10}-1) \tag{3b}$$

$$= 2^x - 1\,. \tag{3c}$$

Hence $x = 10$.