Math Diversion Problem 119

P. Reany

January 24, 2025

> You cannot ask us to take sides against arithmetic.
> — Winston Churchill

The YouTube video is found at:

```
Source: https://www.youtube.com/watch?v=yGmVyOFZdXY
Title: How to Solve Algebraic Equations
 Presenter: Maths Black Board
```

1 The Problem

Given the relations

$$a^2 - ab = 14\,, \tag{1a}$$

$$b^2 + ab = 60\,, \tag{1b}$$

find the values of a, b.

2 The Solution

We begin by adding (1a) and (1b) together, to get

$$a^2 + b^2 = 74\,. \tag{2}$$

Solving (1a) for b:

$$b = \frac{a^2 - 14}{a}\,, \tag{3}$$

and then substituting this value into (2), we get

$$a^2 + \left[\frac{a^2 - 14}{a}\right]^2 = 74\,. \tag{4}$$

Putting this in standard form, we have that

$$a^4 - 51a^2 + 98 = 0\,, \tag{5}$$

with roots for a:

$$a = \pm\sqrt{2} \quad \text{and} \quad a = \pm 7 \tag{6}$$

and with corresponding b values of

$$b = \mp 6\sqrt{2} \quad \text{and} \quad b = \pm 5\,. \tag{7}$$