# Math Diversion Problem 124

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January 24, 2025

> Every great developer you know got there by solving
> problems they were unqualified to solve
> until they actually did it.
> — Patrick McKenzie

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=nJwQGTPIyUM
Title: A Nice Math Olympiad Exponential Equation X^x^2 = 16
 Presenter: MrMath
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## 1 The Problem

Given the relation

$$x^{x^2} = 16\,, \tag{1}$$

find the real values of $x$.

## 2 The Solution

The value 16 on the RHS is not a big number for the problem we have. We could (and maybe should) just start trying low integer values for $x$. Doing that, we quickly find that $x = 2$ is a solution, but then $x = -2$ is also a solution.

On the other hand, a more systematic approach could begin by making the variable substitution

$$x = 2^y \quad (\text{for } x > 0)\,, \tag{2}$$

which would give us

$$2^{y2^{2y}} = 2^4 = 2^{2^2}\,. \tag{3}$$

But on demanding equal exponents and using some algebra, yields:

$$y2^{2y-2} = 1\,. \tag{4}$$

At this point, a reasonable guess is $y = 1$, making $x = 2$. But that's the solution for positive $x$. Clearly, we also have $x = -2$ as a solution.