

Math Diversion Problem 125

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The Axiom of Choice is obviously true; the Well Ordering
Principle is obviously false; and who can tell
about Zorn's Lemma?
— Jerry Bona

The YouTube video is found at:

Source: ?

Title: ?

Presenter: ?

1 The Problem

Given the relation

$$x = \frac{15}{2 + \frac{15}{2 + \frac{15}{2 + \dots}}}, \quad (1)$$

find the real values of x .

2 The Solution

So, we're dealing with an infinitely continued fraction. But this continued fraction is special because a subpart of it is repeated as the entire thing. The best way to show this is to just do it.

$$x = \frac{15}{2 + x}, \quad (2)$$

Putting this in standard form, we have

$$x^2 + 2x - 15 = 0, \quad (3)$$

with possible solutions

$$x = 3 \quad \text{or} \quad x = -5. \quad (4)$$

But we can retain only the positive solution

$$x = 3. \quad (5)$$