

Math Diversion Problem 127

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You either get control of your lusts and feelings of
entitlement, or they will get control of you.

— The Author

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=Uqk2MS5Lcsk>

Title: Math Olympiad | Algebra Problem

Presenter: MathsFocus

1 The Problem

Given the relation

$$\frac{2^{33} + 2^{22} + 2^{11}}{2^{33} - 1}, \quad (1)$$

find its value.

2 The Solution

If you don't already know about the geometric series, this is a good time to learn. Consider the series

$$1 + r + r^2 + r^3 + \dots + r^n, \quad (2)$$

where r is the ratio one any term to its previous term. This series has the closed form value

$$\sum_{i=0}^n r^i = \frac{r^{n+1} - 1}{r - 1} \quad (r \neq 1). \quad (3)$$

This finite series will always converge. If it's infinite, it will always converge over complex or real r if $|r| < 1$.

So, let's return to (1):

$$\frac{2^{33} + 2^{22} + 2^{11}}{2^{33} - 1} = \frac{2^{11}}{2^{33} - 1} [(2^{11})^2 + 2^{11} + 1]. \quad (4)$$

Here $r = 2^{11}$ and $n = 2$,

$$(2^{11})^2 + 2^{11} + 1 = \frac{(2^{11})^3 - 1}{2^{11} - 1} = \frac{2^{33} - 1}{2^{11} - 1}. \quad (5)$$

On plugging this value into (4), we have that

$$\frac{2^{33} + 2^{22} + 2^{11}}{2^{33} - 1} = \frac{2^{11}}{2^{33} - 1} \frac{2^{33} - 1}{2^{11} - 1} = \frac{2^{11}}{2^{11} - 1} = \frac{2048}{2047}. \quad (6)$$