Math Diversion Problem 133

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Abstract

Here we use the unipodal algebra to assist in solving the problem, which is given to us on YouTube. Although I'm referring to the series under the name 'olympiad', the problems are from diverse sources as olympiads, entrance exams, SATs, and the like.

> You cannot read mathematics the way you read a novel. If you zip through a page in less than an hour, you are probably going too fast.
>
> — Sheldon Axler
> (from *Linear Algebra Done Right*)

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=XXFNxw-2bo8&list
                =PLQG1LDIiB85gGzcpH0OP_riNifgU_ZOQW&index=22
Title: A Nice Algebra Equation
       Cambridge University ?
 Presenter: MathMinds
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1 The Problem

Given the relation

$$x + \sqrt{x} = x\sqrt{x}\,, \tag{1}$$

find the values of x over the positive reals.

2 The Solution

Now, my instinct is to rewrite the given equation to this

$$x = \sqrt{x}(x-1)\,, \tag{2}$$

and then to square both sides

$$x^2 = x(x-1)^2\,. \tag{3}$$

In standard form this becomes

$$x^2 - 3x + 1 = 0\,, \tag{4}$$

which has roots

$$x = \frac{3 \pm \sqrt{5}}{2}\,. \tag{5}$$

However, we have to throw out the root $(3-\sqrt{5})/2$ because it would violate the (2) by making $(x-1) = (3-\sqrt{5})/2 - 1 < 0$. We have the additional root of $x = 0$, as is evident from (1). And that makes a total of three solutions.

Note: WolframAlpha also did not include the third root, and at first I did not understand why. Though it's true that $(3-\sqrt{5})/2 > 0$, the problem lay elsewhere, as was demonstrated.