Math Diversion Problem 140

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January 24, 2025

Abstract

Here we use the unipodal algebra to assist in solving the problem, which is given to us on YouTube. Although I'm referring to the series under the name 'olympiad', the problems are from diverse sources as olympiads, entrance exams, SATs, and the like.

> You have to know what to look for, so you can spot it.
> — Papago Indian drug-enforcement border scout

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=Klef16FHNK4
Title: Japanese | Can you solve this?
 Presenter: Master T Maths Class
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1 The Problem

Given the relation

$$4^x + 25^x = 10^{x+1}\,, \tag{1}$$

find the values of x over the real numbers.

2 The Solution

The first thing I did was to simplify a bit:

$$4^x + 25^x = 10 \cdot 10^x\,. \tag{2}$$

Then, if we divide through by 4^x, we can replace a term with variable by a constant.

$$1 + \left(\frac{25}{4}\right)^x = 10 \cdot \left(\frac{10}{4}\right)^x\,, \tag{3}$$

which simplifies to

$$1 + \left(\frac{5}{2}\right)^{2x} = 10 \cdot \left(\frac{5}{2}\right)^{x} . \tag{4}$$

Next, we introduce the variable y, as

$$y \equiv \left(\frac{5}{2}\right)^{x} , \tag{5}$$

by which we can write (4) as

$$y^2 - 10y + 1 = 0 \, , \tag{6}$$

which has roots

$$y = 5 \pm 2\sqrt{6} \, . \tag{7}$$

Finally, this gives us for x:

$$x_{(+,-)} = \frac{\log\left(5 \pm 2\sqrt{6}\right)}{\log\left(5/2\right)} \, . \tag{8}$$