Math Diversion Problem 145

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You don't understand anything until you learn
it more than one way.
— Marvin Minsky

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=7zwAqqH6C58&list
      =PLMvuVeOn1Hd_KIT-dsvIVluQQN3pJrlmX&index=691
Title: Nice Algebra Math Simplification
 Presenter: Master T Maths Class
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1 The Problem

Given the relation

$$\left(\frac{x}{5}\right)^x = 5^{5^2}\,, \tag{1}$$

find the values of x over the real numbers.

2 The Solution

So, I'll make my usual variable transformation in this situation.

$$x = 5^\alpha\,. \tag{2}$$

Then

$$5^{(\alpha-1)5^\alpha} = 5^{5^2}\,, \tag{3}$$

On equating exponents, we have (with some manipulation) that

$$(\alpha-1)5^{\alpha-2} = 1\,. \tag{4}$$

The obvious solution to this is $\alpha = 2$, thus

$$x = 5^2 = 25\,. \tag{5}$$