

# Math Diversion Problem 148

P. Reany

January 24, 2025

You cannot read mathematics the way you read a novel. If you zip through a page in less than an hour, you are probably going too fast.  
— Sheldon Axler  
(from *Linear Algebra Done Right*)

The YouTube video is found at:

Source: [https://www.youtube.com/watch?v=X5iYlxCwGRo&list=PLMvuVe0n1Hd\\_KIT-dsvIVluQQN3pJrlmX&index=46](https://www.youtube.com/watch?v=X5iYlxCwGRo&list=PLMvuVe0n1Hd_KIT-dsvIVluQQN3pJrlmX&index=46)  
Title: A Nice Exponential Equation  
Presenter: Master T Maths Class

## 1 The Problem

Given the relation

$$x^{x^{1+x}} = 256, \tag{1}$$

find the values of  $x$  over the real numbers.

## 2 The Solution

Since the numbers are pretty small, one might save time by guessing. But let's be systematic. First, let's rewrite the given equation as

$$x^{x^{1+x}} = 2^8, \tag{2}$$

So, I'll make my usual variable transformation in this situation.

$$x = 2^\alpha. \tag{3}$$

Then

$$(2^\alpha)^{(2^\alpha)^{(1+2^\alpha)}} = 2^8, \tag{4}$$

which condenses down to

$$2^{\alpha 2^{\alpha(1+2^{\alpha})}} = 2^8. \quad (5)$$

Equating exponents,

$$\alpha 2^{\alpha(1+2^{\alpha})} = 8 = 2^3, \quad (6)$$

with possible integer solution  $\alpha = 1$ . Therefore,

$$x = 2. \quad (7)$$