# Math Diversion Problem 163

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> The definition of a good mathematical problem is the mathematics
> it generates rather than the problem itself.
> — Andrew Wiles

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=9ljcSSCP8oE
Title: A Nice Algebra Problem | Math Olympiad
 Presenter: SALogic
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## 1 The Problem

Given the relation

$$2^x - 9^x = \sqrt{18^x - 81^x}\,, \tag{1}$$

find the values of $x$ over the real numbers.

## 2 The Solution

The first thing to note is that $x = 0$ is a solution.

The given relation can be factored as thus

$$2^x - 9^x = \sqrt{9^x(2^x - 9^x)} = \sqrt{9^x}\sqrt{2^x - 9^x}\,, \tag{2}$$

On squaring both sides and simplifying, we get

$$2^{x-1} = 9^x\,. \tag{3}$$

Since we are not going to be able to solve this with rational values of $x$, we need to take logarithms.

$$(x-1)\log 2 = x \log 9\,. \tag{4}$$

On solving for $x$, we have that

$$x = \frac{\log 2}{\log(2/9)} = -\frac{\log 2}{\log(9/2)}\,. \tag{5}$$

(WolframAlpha disagrees we any solution other than zero.)