

Math Diversion Problem 165

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I love it when a plan comes together.
— Hannibal Smith, *The A-Team*

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=fSf1-Ch1ujI>
Title: Japanese | Can you solve this?
Presenter: Master T Maths Class

1 The Problem

Given the relation

$$\log_2 x = \log_x 4, \tag{1}$$

find the values of x over the real numbers.

2 The Solution

Lemma: A Rule of Logarithms:

$$\log_a b = \frac{\log b}{\log a}. \tag{2}$$

Using the logarithm rule above, (1) becomes

$$\frac{\log x}{\log 2} = \frac{\log 4}{\log x} = \frac{2 \log 2}{\log x}, \tag{3}$$

which then gives us

$$(\log x)^2 = 2(\log 2)^2, \tag{4}$$

$$\log x = \pm\sqrt{2} \log 2, \tag{5}$$

and thus

$$x = 2^{\pm\sqrt{2}}. \tag{6}$$