Math Diversion Problem 169

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People often overlook the obvious.
— Doctor Who

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=MAvbLgfzcww
Title: Nice Exponent Math Simplification
 Presenter: Master T Maths Class
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1 The Problem

Given the relation

$$x^x = 3^{2x+27}\,, \tag{1}$$

find the values of x over the real numbers.

2 The Solution

I'll start with my usual change of variable:

$$x = 3^\alpha\,. \tag{2}$$

On substituting this into (1), we have that

$$(3^\alpha)^{3^\alpha} = 3^{2(3^\alpha)+27}\,, \tag{3}$$

which becomes

$$3^{\alpha\cdot 3^\alpha} = 3^{2(3^\alpha)+3^3}\,. \tag{4}$$

On equating exponents, we get

$$\alpha\cdot 3^\alpha = 2(3^\alpha) + 3^3\,. \tag{5}$$

After trying various small α, I got $\alpha = 3$ to work. Therefore,

$$x = 27\,. \tag{6}$$