# Math Diversion Problem 188

P. Reany

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> Zathras is used to being beast of burden to other
> people's needs. Very sad life... Probably have
> very sad death. But, at least
> there is symmetry.
> —Zathras
> A character from *Babylon 5*

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=Pj7ju4ebtY0
Title: Can We Solve A Transcendental Equation
 Presenter: SyberMath
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## 1 The Problem

Given the relation

$$e^x + x + 1 = 0\,, \tag{1}$$

find the values of $x$.

## 2 The Preparation

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I intend to use the Lambert $W$ function, which goes as follows: If

$$ze^z = B\,, \tag{2}$$

then

$$z = W(B)\,, \tag{3}$$

where there are domain constraints on $B$ that we won't go into here. Warning: This can be a complicated (multi-valued) function to deal with.

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## 3 The Solution

We can rewrite (1) to

$$e^x = -x - 1\,, \tag{4}$$

and prepare for a variable change. Let

$$y = -x - 1\,, \tag{5}$$

and so

$$x = -y - 1\,, \tag{6}$$

Applying this to (4), we get

$$y = e^{-y-1} = e^{-y}e^{-1}\,. \tag{7}$$

A little algebra brings us to

$$ye^y = e^{-1}\,. \tag{8}$$

Applying the rule for the Lambert $W$ function:

$$h = W(he^h)\,, \tag{9}$$

we have that

$$y = W(ye^y) = W(\frac{1}{e})\,. \tag{10}$$

Thus

$$x = -y - 1 = -W(\frac{1}{e}) - 1\,. \tag{11}$$