Math Diversions, Problem 35

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> Success is failing 19 times and succeeding the 20th.
> — Julia Andrews

1 Problem

The YouTube video is found at:

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https://www.youtube.com/watch?v=C5HTplGs1AQ
 Titled: Evaluating A Nice Polynomial | Math Olympiads
 Presenter: SyberMath
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Given the relation

$$b^3 - b = 1\,. \tag{1}$$

find

$$b^5 - b^4\,. \tag{2}$$

2 Solution

From (1) we have that

$$b^3 = b + 1\,. \tag{3}$$

Then,

$$\begin{aligned} b^5 - b^4 &= b^2(b^3) - b(b^3) \\ &= b^2(b+1) - b(b+1) \\ &= b^3 + b^2 - b^2 - b \\ &= b^3 - b \\ &= 1\,. \end{aligned} \tag{4}$$