

# Math Diversions, Problem 39

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It is clear that the chief end of mathematical study  
must be to make the students think.  
— John Wesley Young  
(So think!)

## 1 Problem

The YouTube video is found at:

[https://www.youtube.com/watch?v=GX7MzC0\\_2oM](https://www.youtube.com/watch?v=GX7MzC0_2oM)  
Titled: Harvard University Aptitude Test Tricks  
|| Algebra Problem  
Presenter: Super Academy

Given the relations

$$5^x + 5^y = 15750, \quad (1a)$$

$$x + y = 9, \quad (1b)$$

find the value of  $x$  and  $y$ .

## 2 Solution

Given all the number of 5's in (1a), it seemed reasonable to me to see how many factors of 5 are in 15,750. I guessed there were three and that was correct.

$$15,750 = 5^3 \cdot 126. \quad (2)$$

Thus, (1a) becomes

$$5^{x-3} + 5^{y-3} = 126. \quad (3)$$

Given that we are working with integers, at this point in the analysis, we are supposed to analyze how we can get the sum of powers of 5's to equal an even number that does not end in 0, which wouldn't help anyways. Odd + odd is even, but in this case, one of those odds must be unity.

Without loss of generality, let that be because we choose  $x - 3 = 0$ . Since  $x = 3$ , then from (1b),  $y = 9 - 3 = 6$ . The only thing left to do is to try these values to see if they actually work:

$$5^3 + 5^6 = 125 + 15,625 = 15,750. \quad (4)$$