Math Diversions, Problem 40

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> Algebraic geometry seems to have acquired the reputation of
> being esoteric, exclusive, and very abstract, with
> adherents who are secretly plotting to take over
> all the rest of mathematics. In one respect
> this last point is accurate.
> — David Mumford
> [Not if the Category theorists have
> have anything to do about it.]

1 Problem

The YouTube video is found at:

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https://www.youtube.com/watch?v=aYeuix9nGG0
  Titled: Can you pass College Entrance Aptitude Test ?
        || Find x=?
  Presenter: Super Academy
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Given the relation

$$x^{\log_3 x} = 81\,, \tag{1}$$

find the value of x.

2 Solution

These 'olympiad' type problems are best solved when the student is knowledgeable of powers of 2, 3, and 5, at a minimum. Let's redo (1):

$$x^{\log_3 x} = 3^4\,. \tag{2}$$

Next, let's take the log base 3 of both sides:

$$\log_3 x^{\log_3 x} = \log_3 3^4 = 4\,, \tag{3}$$

which becomes

$$(\log_3 x)^2 = 4 = 2^2 \quad \longrightarrow \quad \log_3 x = 2\,. \tag{4}$$

Thus,

$$3^{\log_3 x} = 3^2 = 9\,. \tag{5}$$

And this gives us

$$x = 9\,. \tag{6}$$