

Math Diversions, Problem 54

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Chance favors the prepared mind.
— Louis Pasteur

1 Problem

The YouTube video is found at:

????

Titled: ????

Presenter: ????

Given the relations

$$\log y = \log_x(2x - 5), \quad (1a)$$

$$\log x = \log_y(x + 15), \quad (1b)$$

find the values of x, y .

2 Solution

The Big Rule of Logarithms when changing base is:

$$\log_a b = \frac{\log_c b}{\log_c a}, \quad (2)$$

where c is an arbitrary positive real number.

We can modify our original equations to:

$$\log y = \log_x(2x - 5) = \frac{\log(2x - 5)}{\log x}, \quad (3a)$$

$$\log x = \log_y(x + 15) = \frac{\log(x + 15)}{\log y}. \quad (3b)$$

Thus (3a) and (3b) become

$$\log x \log y = \log (2x - 5), \quad (4a)$$

$$\log y \log x = \log (x + 15). \quad (4b)$$

From these we conclude that

$$\log (2x - 5) = \log (x + 15). \quad (5)$$

Hence,

$$(2x - 5) = (x + 15), \quad (6)$$

and thus

$$x = 20. \quad (7)$$

Solving for y , we get

$$y = \log^{-1} \frac{\log 35}{\log 20}. \quad (8)$$